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TYPE-OF-EMPLOYMENT CHOICE**

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DECLINING SEARCH FRICTIONS AND TYPE-OF-EMPLOYMENT CHOICE*

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Abstract

Do more people choose to become self-employed when search frictions decline? The origin story of the gig economy suggests that improvements in communication technologies increase the self-employment rate, while cross-country evidence suggests the opposite. We reconcile conventional wisdom with the data by introducing frictions in labour *and* goods markets in a new model of self-employment. Declining labour market frictions *decrease* self-employment, while declining goods market frictions *increase* self-employment. We study the impact of the most salient recent reduction in frictions - the roll-out of broadband Internet - in a panel of OECD countries. We find that the effect of declining goods market frictions dominates: the arrival of broadband Internet has halted three quarters of the average downward trend in self-employment rates.

JEL classification: J21, J64, O33

Keywords: Self-employment, goods markets, labour markets, search frictions, Internet, matching efficiency.

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1 Introduction

In the popular press, there is the widespread perception that improvements in information and communication technologies (ICT) have contributed to the rise of own-account work and self-employment. Increased access to broadband Internet has been accompanied by the development of new apps that facilitate self-employed handymen and cab drivers finding customers, supposedly resulting in a “gig economy”.

However, as we will document below, most OECD countries have not experienced any recent increases in self-employment rates. Falling self-employment rates are consistent with new evidence showing that labour market frictions are important to explain cross-country variation in self-employment rates (e.g. Rud and Trapeznikova (2020)). Taking this finding as potential explanation for developments over time, rising payroll employment and falling self-employment can be expected to result from declining labour market frictions due to improvements in ICT. Therefore, we ask: do improvements in ICT increase or decrease self-employment?

We propose a parsimonious theory reconciling that declining search frictions may have both positive and negative effects on the self-employment rate. Our key innovation is to introduce frictional entry to a centralised goods market in addition to a frictional labour market. Ex ante identical, risk-averse workers choose between self-employment and searching for a payroll job. The self-employed face the risk of not selling output on their own, while large firms insure payroll employees against goods market risk: employees obtain labour income even if their production cannot be sold in the goods market. However, searching for a payroll job comes with the risk of unemployment. Moreover, the self-employed are the sole claimants of the fruits of their labour while employees have to share these with their employers.

The composition of employment arises endogenously as a unique mixed-strategy Nash equilibrium of workers indifferent between the two types of employment. To understand the intuition behind this equilibrium, note that employees who can supply their production to the goods market produce more than the self-employed. They produce more because they have to cover the costs of vacancies and of insurance for those employees who cannot supply their production. In compensation, finding a job is more likely than selling production in the goods market in equilibrium. Now consider an off-equilibrium movement of workers from the labour market to self-employment. Because payroll employees produce more, moving workers from the labour market to self-employment acts as a negative supply shock, increasing prices. The self-employed benefit directly from higher prices through an increase in expected profits. However, job searchers benefit more; directly from an increase in wages *and* indirectly via a higher job-finding probability. These general equilibrium effects pull workers back to the labour market.

We model the effects of improvements in ICT as reductions in market frictions: they increase both labour market efficiency and the ease of entry of sellers in the goods market. The main implication of our theory is that the general equilibrium effect of improvements in ICT on the self-employment rate can be of *arbitrary* sign. On the one hand, we show that reductions in goods market frictions unambiguously increase the self-employment rate. Even though large firms benefit equally from reductions in goods market frictions, lower goods market risk decreases the value of insurance that firms offer, increasing the incentives to become

self-employed.

On the other hand, reductions in labour market frictions make searching for a payroll job more attractive and decrease the equilibrium self-employment rate. Improvements in labour market matching efficiency act as a positive supply shock, exerting downward pressure on prices. Lower prices and higher matching efficiency in the labour market discourage workers from choosing self-employment and prompt them to search for jobs at firms. We show that our comparative statics are robust to the introduction of a labour force participation decision, idiosyncratic preferences for, or costs of, self-employment, and long-term employment relationships.

To assess which of the two channels dominates, we estimate the effect of the most salient recent reduction of goods and labour market frictions, the introduction of broadband Internet, on self-employment. Since our theory is concerned with general equilibrium effects, we use variation in broadband access across a panel of OECD countries. To account for the possible endogeneity of broadband Internet, we instrument broadband adoption by a logistic diffusion model in which the availability of pre-existing technologies predicts broadband penetration, just as in Czernich, Falck, Kretschmer, and Woessmann (2011).

We find that broadband Internet prompts more self-employment and that this effect is quantitatively important. Our findings lend support to the interpretation that improvements in communication technologies have increased the incentives to become self-employed. In the light of our model, we infer that the general equilibrium effect of broadband Internet on the self-employment rate resulted from the goods market channel dominating the labour market channel.

We make three contributions to the literature. Firstly, we propose a novel, tractable model of selection into self-employment, one that puts technology and market frictions at the center stage. In fact, our model is orthogonal to the large body of earlier work summarized succinctly in Parker (2004). This literature focuses on individual heterogeneity as the main determinant of becoming self-employed. For instance, Lucas (1978), Jovanovic (1982) and Poschke (2013) assume that being self-employed requires a separate skill, while De Meza and Southey (1996) find that self-employed entrepreneurs are simply more optimistic than employees. Our theory relies on risk aversion, similar to Kihlstrom and Laffont (1979). However, unlike theirs, our theory does not rely on any ex-ante differences among individuals. Similar to Rissman (2003), self-employment is an alternative to searching for a job in our model. However, she assumes returns to self-employment are drawn from an exogenous distribution riskier than the wage distribution. By introducing goods market frictions and large firms providing insurance against income risk, we offer a model that endogenously generates those risk differentials.¹

Our second contribution concerns the effects of modern technologies on the composition of employment. Preceding the rise of the gig economy, Blau (1987) and Fairlie (2006) showed the importance of technological change, and personal computers in particular, to explain U.S. trends in self-employment. We follow Kolko (2012), Akerman, Gaarder, and Mogstad (2015), Audretsch, Heger, and Veith (2015), Hjort and Poulsen (2019) and Czernich et al. (2011) in

¹ Other papers that study the macroeconomic consequences of goods market frictions (e.g. Kaplan and Menzio (2016), Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2015)) do not consider self-employment.

focusing on the impact of broadband, but investigate its effect on self-employment. Although economic development has generally been associated with decreases in self-employment rates, we find that the arrival of broadband Internet has halted three quarters of the average downward trend in self-employment rates. This finding is robust to the inclusion of important institutional variables.²

Our third contribution concerns the comparative statics of declining search frictions on the labour market. Closely related to our theoretical framework are therefore papers that address the joint determination of payroll employment, unemployment and self-employment rates. Poschke (2019) and Feng, Lagakos, and Rauch (2018) explain variations in employment status across a wide range of countries, while Rud and Trapeznikova (2020) focus on Sub-Saharan Africa. All of these papers stress the importance of labour market frictions, as we do, but neither considers goods market frictions.³ Allowing for type-of-employment choice and goods market frictions qualifies the standard results on the effects of increases in matching efficiency on the job-finding probability and unemployment. When matching efficiency increases, labour market tightness falls because of congestion caused by workers leaving self-employment. This insight offers a new perspective on the puzzle, identified by Martellini and Menzio (2020), why advancements in ICT, prompting higher matching efficiency in the labour market, did not result in falling unemployment. In our model, only reductions in goods market frictions unambiguously increase the job-finding probability. Better matching technology in the labour market may even decrease the job-finding probability when the decrease in self-employment is sufficiently strong.

2 Model

We consider a stylised one-period economy inhabited by three types of agents - consumers, workers and firms - that interact in two markets: the goods market and the labour market. As in Rudanko (2009), workers are risk-averse and hand-to-mouth and can only be insured against income risk in payroll employment at large, risk-neutral firms.⁴ We focus on the type-of-employment choice and the risks associated with self-employment, in addition to the risks of entering the labour market.

The structure of the model is illustrated in Figure 1. Workers face a choice between self-employment and payroll-employment. When workers choose to become self-employed, they go to the goods market directly, where they face the risk of not being able to sell their production. If, on the other hand, workers choose to seek employment in a firm, they face the risk of unemployment. Once hired, however, the employee is insured by the firm against the risk in the goods market. The firms create an endogenously determined measure of vacancies and are able

² Studies that seek to explain cross-country differences in self-employment rates and focus on institutional variation include Acs, Audretsch, and Evans (1994), Blanchflower (2000), Parker and Robson (2004), Robson and Wren (1999), Staber and Bögenhold (1993), Torrini (2005), and Kumar (2012).

³ Other papers model and estimate the flows across payroll, self- and unemployment in a single country, and assess counterfactual policies (e.g. Bradley (2016) and Narita (2020)), or business cycles (e.g. Visschers, Millan, and Kredler (2014)).

⁴ Rudanko (2009) uses slightly different nomenclature. In her model, risk neutral entrepreneurs can open a continuum of single-worker production units which she refers to as firms. We have large firms that hire a measure of workers, each staffing a single-worker production unit.

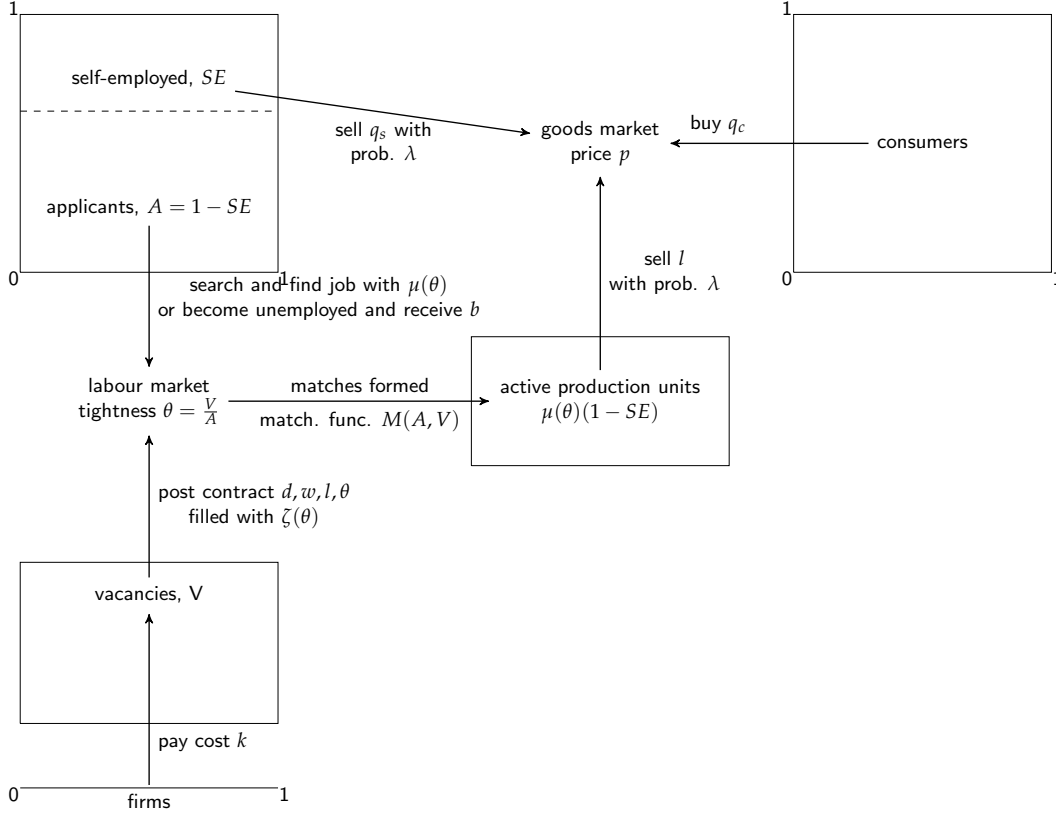


Figure 1: Overview of the model.

to offer insurance to employees by the virtue of their large size. We formalize this description and present the relevant details below.

2.1 Model setup

Consumers Consumers live on a unit square $[0, 1] \times [0, 1]$ and freely enter a perfectly competitive goods market. Their preferences over the consumption good are captured by a linear utility function. To acquire q_c units of the consumption good at prevailing price p , they pay with a spot good – the numéraire – that they produce according to a strictly convex production function g . Thus, given the market price p , the utility of consuming q_c reads:

$$V^B(q_c; p) = q_c - g(pq_c).$$

The optimal demand q_c is therefore pinned down by the following equation:

$$g'(pq_c) = \frac{1}{p}. \quad (1)$$

Note, as the total mass of buyers is equal to 1, this is also the aggregate demand equation.

Workers and Career Choice Workers live on a unit square $[0, 1] \times [0, 1]$. They produce the consumption good desired by consumers at linear cost, and value the spot good supplied by consumers. In particular, they have a strictly increasing and strictly concave utility of

consumption $u(c)$ so that $u'(c) > 0$ and $u''(c) < 0$. We also normalize $u(0) = 0$.

The workers face a career choice: they can either become self-employed which yields expected utility V^{SE} , or they can enter the labour market which yields expected utility V^{LM} . Let $0 \leq SE \leq 1$ be the share of workers who become self-employed, then $A = 1 - SE$ is the measure of job applicants in the labour market hoping to find a job at a firm. Thus, we rule out job search during self-employment. Assuming that the type-of-employment choice is exclusive is simply the extreme version of the assumption that running a viable business (that actually results in earnings) takes time and reduces job search intensity.

Firms and Labour Market Firms live on a unit segment $[0, 1]$. The labour market is characterized by search frictions and there is free entry of vacancies. The market is segmented into sub-markets, indexed with i , distinguished by the terms of trade. Each firm h opens a measure of vacancies v_i^h in submarket i at a cost of k units of the numéraire per vacancy. A filled vacancy becomes a one-worker production unit. Each applicant observes the terms of trade in each sub-market and directs their search to one. The overall stock of vacancies in a sub-market i is $V_i = \int_0^1 v_i^h dh$.

The total number of matches between applicants A_i and vacancies V_i in a sub-market is given by a Cobb-Douglas matching function $M_i = EA_i^\phi V_i^{1-\phi}$ with $\phi \in (0, 1)$. The parameter $E > 0$ captures the exogenous matching efficiency that increases when labour market frictions fall. Let $\theta_i = V_i/A_i$ be the ratio of vacancies to applicants in a sub-market. The probability that an individual vacancy is filled is then $\zeta(\theta_i) = M_i/V_i$. Correspondingly, each applicant finds a job with probability $\mu(\theta_i) = M_i/A_i$.⁵ Using the definitions of the job filling and job finding probabilities, we can also write $\zeta(\theta) = E\hat{\zeta}(\theta)$ and $\mu(\theta) = E\hat{\mu}(\theta)$ with $\hat{\zeta}(\theta) = \theta^{-\phi}$ and $\hat{\mu}(\theta) = \theta^{1-\phi}$ so that $\hat{\zeta}'(\theta) < 0$, $\hat{\zeta}''(\theta) > 0$ and $\hat{\mu}'(\theta) > 0$, $\hat{\mu}''(\theta) < 0$.

Frictional Entry in the Goods Market Each self-employed worker and each one-worker production unit face frictional entry into the competitive goods market: only with probability $\lambda \in (0, 1)$ can they enter and sell their production.⁶ We will also say that when a worker manages to enter the goods market, this worker is visible in the goods market. The motivation for this formulation is that we think of search frictions in the goods market as consumers simply not being aware of sellers. Declining search frictions then allow more sellers to compete for the demand of consumers because they have become visible to them.⁷ We model declining goods market frictions as an increase in λ . Hence, we assume that large firms and self-employed workers benefit equally from reductions in search frictions in the goods market.⁸

⁵ We assume throughout that the constraint that these probabilities cannot exceed 1 is not binding.

⁶ We consider the less tractable case of an endogenous probability to enter the goods market in Appendix C.3. As we show there, the characterisation of equilibrium and the effects of reductions in search frictions on the self-employment rate are not affected by this extension.

⁷ We do not only think of declining search frictions as increasing the likelihood that a seller is visible to a customer, but also as making it more likely that a previously unknown seller is considered sufficiently trustworthy by a customer to do business. Although we do not model such mechanisms explicitly, we do believe that ratings and reviews on platforms have decreased asymmetric information and facilitated trade.

⁸ One can argue that before the arrival of broadband Internet large firms had smaller problems being visible to consumers than self-employed workers, so that the self-employed should benefit more from the arrival of broadband. However, throughout the paper we tie our hands and derive comparative statics for the case in which reductions in goods market frictions do not result in a higher self-employment rate essentially by assumption.

Self-Employment The self-employed who do not become visible in the goods market do not produce and do not earn any income. Thus, the expected value of becoming self-employed and, upon making it to the goods market, supplying q_s , reads:

$$V^{SE}(q_s; p) = \lambda (u(pq_s) - q_s),$$

with the optimal production choice q_s satisfying:

$$u'(pq_s) = \frac{1}{p}. \quad (2)$$

Payroll Employment The law of large numbers applies, so that an individual firm faces no uncertainty. The firm receives expected profits and can commit to pay salaries to employees who were hired but whose production units did not become visible in the goods market. Workers who do not find a job become unemployed and receive unemployment benefits b .

Employment contracts are characterised by greater complexity than spot trades in markets for goods. To allow firms to post optimal contracts, we set up a competitive search environment in the labour market. In this environment, the expected value of going to a sub-market cannot be less than the market utility V^{LM} . Following the standard argument in such models with homogeneous workers and firms, there will be exactly one sub-market active and hence we scrap the i index. The firms post a contract characterised by (θ, w, l, d) in a sub-market with tightness θ that will involve paying d when the production unit is not visible in the goods market, and that otherwise will pay a wage rate w and will require the worker to produce l . The contract solves the constrained optimisation problem to maximize expected per-vacancy profit Π , taking as given the expected utility V^{LM} that applicants obtain in the labour market:

$$\begin{aligned} \max_{l, \theta, w, d} \quad & \zeta(\theta) (\lambda(p - w)l - (1 - \lambda)d) \equiv \Pi \\ \text{subject to: } & \mu(\theta) (\lambda (u(wl) - l) + (1 - \lambda)u(d)) + (1 - \mu(\theta))u(b) \geq V^{LM} \end{aligned}$$

We derive the properties of this contract in Appendix A.1. Notably, the risk-neutral firm finds it optimal to fully insure the risk-averse workers against the goods market income risk:

$$wl = d.$$

Hence, we can scrap d completely such that the expected profit per vacancy and the market utility constraint become, respectively:

$$\begin{aligned} \Pi &= \zeta(\theta) (\lambda p - w)l, \\ \mu(\theta) (u(wl) - \lambda l) + (1 - \mu(\theta))u(b) &\geq V^{LM}. \end{aligned}$$

The employees always receive income wl but produce only when their production unit is visible

in the goods market. The optimal contract, given p , then satisfies the following conditions:

$$u'(wl) = \frac{1}{p}, \quad (3)$$

$$\phi(\lambda p - w)l = (1 - \phi) \frac{u(wl) - \lambda l - u(b)}{u'(wl)}. \quad (4)$$

Furthermore, there is one sub-market with tightness $\theta = V/A = V/(1 - SE)$. As there is free entry of vacancies, firm profits per vacancy must be zero:

$$k = \zeta(\theta)(\lambda p - w)l. \quad (5)$$

2.2 Equilibrium

To close the model we require the goods market to clear. The market-clearing equilibrium price is defined implicitly as a price for which aggregate demand equals aggregate supply. The formal equilibrium concept we are after is a mixed-strategy solution to the career choice, one in which payroll- and self-employment co-exist, pursuing each type of employment yields the same expected utility, and workers randomize between them.

Definition 1 (Mixed-strategy career-choice equilibrium) *A mixed-strategy career-choice equilibrium (MSCC-equilibrium) is a tuple $(SE, p, q_s, q_c, w, l, \theta)$ such that:*

- $0 < SE < 1, 0 < \theta$, both types of employment are chosen in equilibrium and active in the goods market,⁹
- given p , each consumer demands q_c as prescribed by equation (1), each visible self-employed sells q_s given by equation (2) and θ, w, l satisfy equations (3) - (5),
- given θ, w, l, q_c and q_s , p and SE simultaneously clear the goods market and make workers indifferent between self-employment and searching for a job at a firm:

$$\lambda(SEq_s + (1 - SE)\mu(\theta)l) = q_c, \quad (6)$$

$$V^{SE} = \lambda(u(pq_s) - q_s) = \mu(\theta)(u(wl) - \lambda l) + (1 - \mu(\theta))u(b) = V^{LM}. \quad (7)$$

The equilibrium is a vector consisting of 7 variables that jointly solve equations (1) - (7). In an MSCC-equilibrium, workers are indifferent between the two careers and randomize over them. The endogenously determined self-employment rate SE is the weight assigned to self-employment in this randomisation.

3 Analytical results

We proceed by describing the MSCC-equilibrium in our model. We start with some analytical observations assuming an equilibrium exists. Then, we explain how to construct an MSCC-equilibrium. Next, we derive comparative statics with respect to reductions in frictions in the

⁹ Note that with $\theta = 0$ there would be no payroll employment.

goods and in the labour market. Finally, we discuss the role of the main assumptions in our model, and we conclude by presenting three extensions to our framework.

3.1 MSCC equilibrium: characterisation, existence and comparative statics

Lemma 1 *Let $(SE, p, q_s, q_c, w, l, \theta)$ be an MSCC-equilibrium. Then:*

1. *$wl = pq_s$, $w < p$, and $l > q_s$: an employee produces more than a self-employed. For $u(b) = 0$, also $\mu(\theta)l > q_s$: the expected production of an applicant is larger than that of a self-employed.*
2. *There exists a differentiable and strictly increasing function $\psi(p)$ such that the workers' career-choice indifference reads:*

$$V^{SE} = \lambda\psi(p) = u(b) + \mu(\theta)\phi[\psi(p) - u(b)] = V^{LM}, \quad (8)$$

the free-entry condition becomes:

$$k = \zeta(\theta)(1 - \phi)p[\psi(p) - u(b)], \quad (9)$$

and the sharing rule can be written as:

$$\lambda l = (1 - \phi)[\psi(p) - u(b)] + q_s. \quad (10)$$

Furthermore, when $u(b) = 0$, then also $\lambda < \mu(\theta)$: securing income is more likely in payroll than in self-employment.

3. *The demand function is strictly monotone: $dq_c/dp < 0$.*

The proof is provided in Appendix A.2. For workers who secure it, labour income is equalized between the two careers in equilibrium, because the linear cost of effort makes the utility between firms and workers transferable. However, the employees who staff production units that are visible in the goods market produce more per capita than the visible self-employed, for two reasons. First, because they must generate profits to cover the vacancy creation costs, which explains the difference between q_s and λl . Indeed, the expected markup of production by an employee over that of a visible self-employed measured in units of the numéraire exactly covers the expected recruiting costs $k/\zeta(\theta)$ per employee, as merging (9) and (10) reveals:

$$p \underbrace{(\lambda l - q_s)}_{\text{markup}} = \frac{k}{\zeta(\theta)}$$

Second, employees produce more than the self-employed because they pay for the insurance against the goods market risk that all employees obtain, by producing the additional $(1 - \lambda)l$.¹⁰

¹⁰ As we abstract from leisure in the model, q_s and l should be interpreted in terms of production (and not as hours worked) which result from an identical production function for firms and the self-employed. Therefore, the model predicts employees to be more productive per capita than the self-employed at the cost of a higher disutility of effort in payroll employment.

Lemma 1 highlights how the structure of the model simplifies, helping us to reduce the dimensionality of the problem of finding an equilibrium. The linear cost of effort allows us to write the terms of the employment contract solely as a function of p , just as the decisions of consumers and the self-employed. Given the price p we can thus obtain q_c, q_s, l and w . As a result, we are left with three nonlinear equations – workers' indifference (8), free entry of vacancies (9) and goods market clearing (6) – in three unknowns: p, θ and SE . Finally, aggregate demand is strictly decreasing in price p .

However, there are still two remaining obstacles to solving this reduced system of equations. First, through the linear effort assumption, we have taken a particular stand on the separability of the worker's consumption and labour supply. As a result, for some specifications of the utility function $u(c)$, the supply decision of the self-employed (2) might be non-monotone in price p . This non-monotonicity may carry over to the equilibrium level of employee production l by virtue of equation (10). When this happens, aggregate supply – the left-hand side of the goods market clearing condition (6) – becomes non-monotone in price p , which may lead to a multiplicity of equilibria. To rule out this possibility, we make the following assumption.

Assumption 1 (Monotone aggregate supply) *The utility function $u(c)$ is such that $dq_s/dp > 0$.*

This assumption is analogous to the absence of a wealth effect on labour supply, as in the case of GHH preferences. This condition is met by CRRA utility functions of the form $u(c) = A \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma \in (0, 1), A > 0$, which is a common choice in quantitative and applied work using models similar to ours (see e.g. Berentsen, Menzio, and Wright (2011)). Note that through (10), Assumption 1 also guarantees that $dl/dp > 0$.¹¹

The second obstacle is due to the presence of unemployment benefits b . Unemployment insurance (UI) not only affects the division of the surplus between employees and firms, but also the workers' indifference condition unless $u(b) = 0$. The constructive proof of Theorem 1, our first main result, explains that the equilibrium with $u(b) = 0$ exists in a region of the parameter space. The existence of an equilibrium with positive unemployment benefits then boils down to the existence of a solution to an ordinary differential equation with respect to $u(b)$ with an initial condition equal to $u(b) = 0$. We offer more details on this differential equation in Appendix C.1.

Theorem 1 (Existence of equilibrium without UI) *Suppose $u(b) = 0$. Then, there exist numbers $0 < \underline{k}(\lambda, \phi) < \bar{k}(\lambda, \phi) < \infty$ such that an MSCC equilibrium exists and is unique, if $\lambda < \phi$ and $k \in (\underline{k}(\lambda, \phi), \bar{k}(\lambda, \phi))$.*

Proof. When there is no UI, the equilibrium conditions (8) - (9) simplify to:

$$\lambda = \phi\mu(\theta), \quad (11)$$

$$k = \zeta(\theta)(1 - \phi)p\psi(p). \quad (12)$$

For given λ and ϕ , worker's indifference (11) pins down the equilibrium market tightness θ^* , unless λ and ϕ are such that this equation has no solution, meaning that there is only one type

¹¹ Even for preferences that do not satisfy Assumption 1, we could ensure uniqueness of an MSCC-equilibrium by choosing an appropriate buyers' production function g , resulting in an aggregate demand function that crosses supply only once.

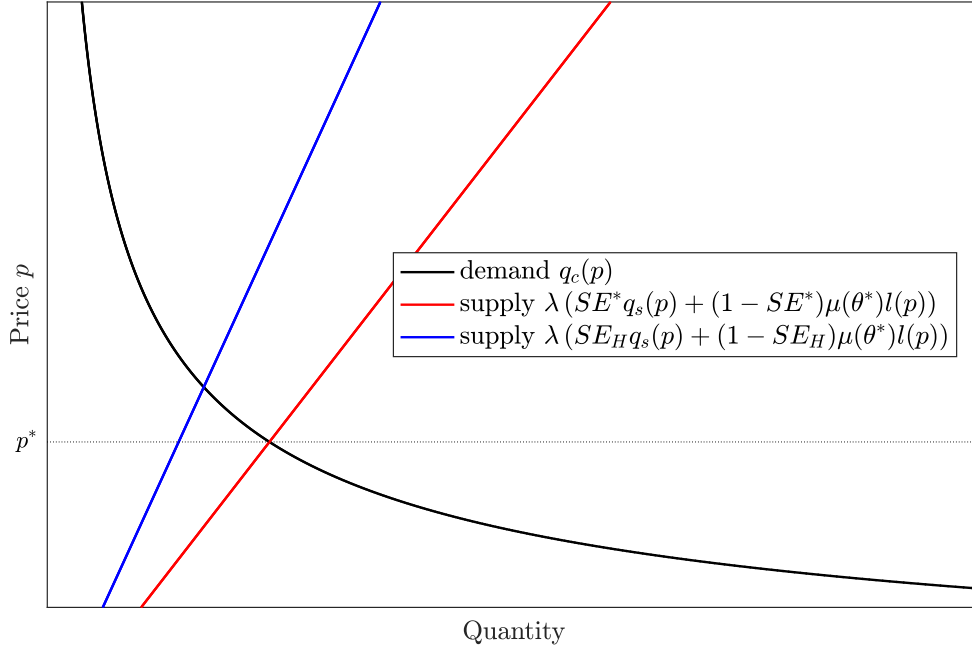


Figure 2: Aggregate supply curves for two levels of the self-employment rate, SE^* and SE_H with $SE^* < SE_H$, crossing the aggregate demand curve, such that the goods market clears for SE^* at an equilibrium price p^* consistent with tightness θ^* pinned down by workers' indifference. Note, the lowest value on the vertical axis is greater than zero.

of employment in equilibrium. Given θ^* , the equilibrium price p^* is pinned down by free-entry condition (12) for a given k . Finally, an MSCC equilibrium exists if, and only if, there exists a self-employment rate $SE^* \in (0, 1)$ such that the goods market clears given p^* and θ^* .

By virtue of Assumption 1 there is at most one self-employment rate that clears the goods market. For a given value of p , the quantities produced q_s and l are fixed, so that moving the self-employment rate amounts to shifting the aggregate supply curve, since Lemma 1 shows that $\mu(\theta)l > q_s$. We illustrate this algebraic route to finding the equilibrium in Figure 2 with two values of the self-employment rate, SE^* and SE_H , with $SE^* < SE_H$. For each self-employment rate, there exists a corresponding aggregate supply curve, for a fixed value of labour market tightness θ^* (pinned down by workers' indifference in (11)) and fixed q_s and l pinned down by the equilibrium price p^* (implied by free entry in (12)).

The bounds $(\underline{k}(\lambda, \phi), \bar{k}(\lambda, \phi))$ on the vacancy posting cost compatible with an MSCC-equilibrium can be found by considering the limiting cases of $SE \mapsto 0^+$ and $SE \mapsto 1^-$. The limiting case of $SE \mapsto 1^-$ yields the highest price level \bar{p} , and $SE \mapsto 0^+$ yields the lowest price level \underline{p} which results in an MSCC-equilibrium for a given θ^* . These bounds map one-to-one to bounds on the vacancy-creation cost k , ensuring that the price level that clears the goods market is consistent with $0 < SE < 1$. Therefore, for sufficiently high values of k there will be no payroll employment, and for sufficiently low k there might be no self-employment. Furthermore, the larger λ and ϕ are, the lower the values of the vacancy-creation cost k that are consistent with existence of the equilibrium. In Appendix C.2 we solve for an equilibrium for one particular

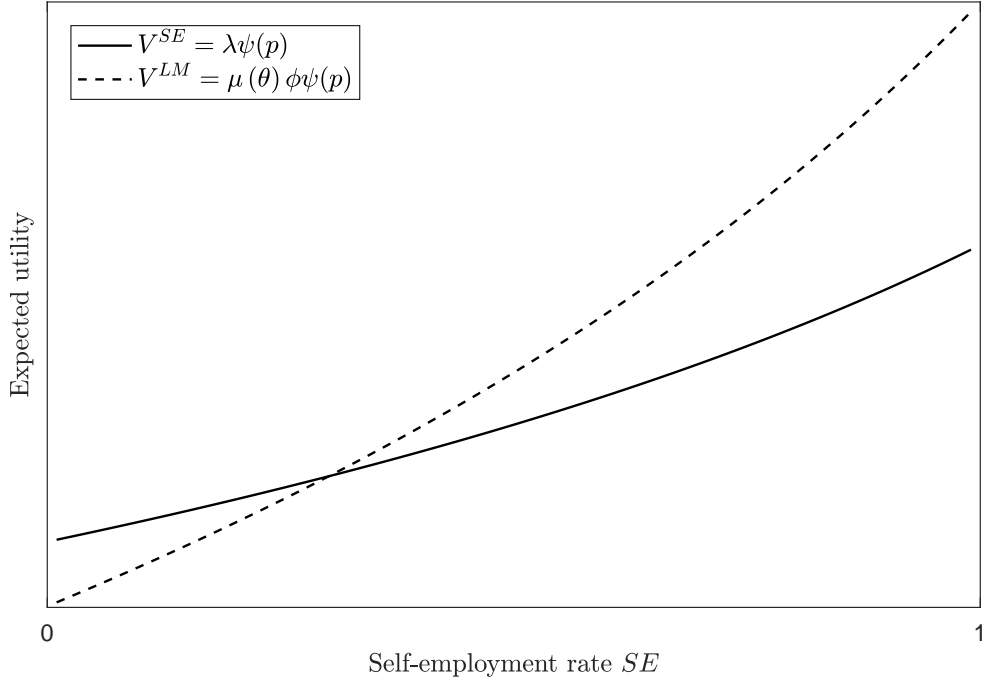


Figure 3: Expected utility of each career – entering self-employment or the labour market – as a function of the self-employment rate, while the goods market clears and free entry of vacancies holds for all self-employment rates. Figure created with $u(c) = 2\sqrt{c}$ and $b = 0$.

example of the matching and utility functions. ■

This description of how to construct an MSCC-equilibrium starts with the premise that workers are indifferent between careers. To understand the economic intuition behind the equilibrating forces in the model, consider taking some workers away from the labour market and making them enter self-employment instead, while keeping the free-entry and goods-market clearing conditions satisfied. When more workers enter self-employment, they decrease congestion in the labour market, which benefits applicants. The free-entry condition requires that firms are compensated for a tighter labour market with a higher price, which increases the surplus per filled vacancy. The price rises because the self-employed produce less than applicants. As a result, increasing the self-employment rate benefits job applicants via two channels. First, a higher self-employment rate increases the price and thus the surplus to be shared with employees. Second, the remaining applicants enjoy a greater likelihood of finding a job. The self-employed benefit only from the first effect. Formally, these effects of the self-employment rate on the incentives to enter self-employment rather than the labour market are:

$$\frac{d}{dSE} (V^{SE} - V^{LM}) = \underbrace{\lambda\psi'(p) \frac{dp}{dSE} - \mu(\theta)\phi\psi'(p) \frac{dp}{dSE}}_{\text{net price effect}} - \underbrace{\mu'(\theta) \frac{d\theta}{dSE} \phi\psi(p)}_{\text{tightness effect}} < 0. \quad (13)$$

However, as in any equilibrium $\lambda = \phi\mu(\theta)$, the self-employed and applicants benefit equally from the change in prices and the net price effect on workers' indifference is nil. Consequently, the effect of an increase in the job-finding rate is decisive and pulls workers back to the labour

market. We illustrate these considerations in Figure 3. When $SE > SE^*$, the expected utility of becoming an applicant exceeds that of pursuing self-employment, $V^{SE} < V^{LM}$, pulling workers into the labour market. The converse is true for $SE < SE^*$.

Our second main result is on the comparative statics of the self-employment rate with respect to matching efficiency in the labour market E and ease of entry in the goods market λ . Even though we rely mostly on the special case of zero unemployment benefits when explaining the intuition behind the results below, we show in the proofs that the results also hold for $u(b) > 0$.

Theorem 2 (Reductions in frictions and the self-employment rate) *In any MSCC-equilibrium, a higher probability of selling in the goods market increases the equilibrium self-employment rate, $dSE/d\lambda > 0$, and increases the job-finding probability, $d\mu(\theta)/d\lambda > 0$. Improvements in matching efficiency in the labour market decrease the self-employment rate, $dSE/dE < 0$, and at least weakly decrease the job-finding probability, $d\mu(\theta)/dE \leq 0$.*

The proof is provided in Appendix A.3. The direct effect of an increase in λ is to increase the likelihood of securing income in self-employment. For workers to remain indifferent, the job-finding probability must also increase, which requires labour market tightness to rise. A higher market tightness decreases the job-filling rate, and for the free-entry condition to remain satisfied, prices need to rise. Prices need to rise because due to the insurance in the equilibrium contract, firms do not directly benefit from increases in λ , as can be seen in (9). In equilibrium, higher prices and a higher labour market tightness result from a higher self-employment rate, since the self-employed produce less and do not congest the labour market. As a result, an increase in λ increases the equilibrium self-employment rate *and* the job-finding probability.

When labour market matching efficiency E increases, it raises the job-filling probability. Firms open more vacancies, attracting workers from self-employment. However, when $u(b) = 0$, the equilibrium market tightness is fixed by worker's indifference in (11): the equilibrium job finding probability $\mu(\theta)$ cannot change when λ remains fixed. Hence, any increase in matching efficiency E must be exactly offset by adjustment in θ , which happens by the inflow of workers from self-employment. When $u(b) > 0$, the motive to abandon self-employment is even stronger. In this case, the congestion created by the inflow of workers from self-employment overturns the effects of the initial increase in E . Consequently, an increase in matching efficiency decreases the self-employment rate and cannot increase the job-finding probability. In equilibrium, prices fall because payroll employment is higher.

As a result, our model offers a new perspective on the puzzle identified by Martellini and Menzio (2020). They wonder why unemployment does not show a trend despite apparent improvements in labour market efficiency. In our model, because of type-of-employment choice and goods market frictions, better matching technology in the labour market does not increase the job-finding probability, while reductions in goods market frictions do.¹²

¹² The effect of a better matching technology in the labour market on the job-finding probability depends to a certain extent on the assumption of an exogenous probability to enter the goods market. We show in Appendix C.3 that making this probability endogenous introduces an additional effect that exerts upward pressure on the job-finding probability.

3.2 Discussion

To further explain the workings of the model, we turn to its two main assumptions. The first assumption is that large firms insure workers against goods market risk. The second assumption is that workers can freely choose their type of employment.

3.2.1 Importance of income risk

Our model of type-of-employment choice relies on differences in the exposure to risk in payroll- and self-employment. We show here that these differences are the *raison d'être* of an MSCC-equilibrium. In the baseline model, firms enjoy the benefits of the law of large numbers. By the virtue of their size, they can guarantee income also to workers staffing production units that did not become visible in the goods market. Now suppose that firms were one-worker firms instead, so that they would only be able to pay a wage upon being visible in the goods market. The optimal contract-posting problem would then read:

$$\begin{aligned} & \max_{\theta, w, l} \zeta(\theta) \lambda (p - w) l \\ & \text{subject to: } \mu(\theta) \lambda (u(wl) - l) + (1 - \mu(\theta)) u(b) \geq V^{LM}. \end{aligned}$$

The optimality conditions for the employment contract, which we derive in Appendix A.4, are:

$$u'(wl) = \frac{1}{p}, \quad (14)$$

$$\phi \lambda (p - w) l = (1 - \phi) \frac{\lambda (u(wl) - l) - u(b)}{u'(wl)}, \quad (15)$$

while the adjusted free-entry condition is:

$$k = \zeta(\theta) \lambda (p - w) l. \quad (16)$$

Given these conditions, we can now provide the definition of a mixed-strategy career-choice equilibrium adjusted to one-worker firms.

Definition 2 (MSCC-equilibrium with one-worker firms) *A mixed-strategy career-choice equilibrium with 1-worker firms is a tuple $(SE, p, q_s, q_c, w, l, \theta)$ such that:*

- $0 < SE < 1, 0 < \theta$, both types of employment are chosen in equilibrium and active in the goods market,
- given p , each consumer demands q_c as prescribed by equation (1), each self-employed sells q_s given by equation (2) and θ, w, l satisfy equations (14) - (16) given p ,
- given θ, w, l, q_c and q_s , p and SE simultaneously clear the goods market as in (6) and make workers indifferent between self-employment and searching for a job at a firm:

$$V^{SE} = \lambda (u(pq_s) - q_s) = \mu(\theta) \lambda (u(wl) - l) + (1 - \mu(\theta)) u(b) = V^{LM}. \quad (17)$$

It turns out that such an equilibrium could only exist under knife-edge conditions that make the type-of-employment choice ill-defined.

Proposition 1 *If an MSCC-equilibrium with one-worker firms exists, then $k = 0$. Not only the incomes in the two types of employment are equalised, $wl = pq_s$, but so are the quantities produced, $l = q_s$, so that wages equal prices, $w = p$. Workers are indifferent not only between self-employment and entering the labour market, but also between payroll- and unemployment.*

Proof. We provide the sketch of the main features of this equilibrium here, and fill in the remaining details in Appendix A.4. We find that the workers' indifference condition and the free-entry condition, respectively, become:

$$\lambda\psi(p) = u(b), \quad (18)$$

$$k = \zeta(\theta)(1 - \phi)p[\lambda\psi(p) - u(b)]. \quad (19)$$

The workers' indifference condition (18) means that the expected value of self-employment, $V^{SE} = \lambda\psi(p)$, must be equal to that of receiving (positive) unemployment insurance with certainty, $V^{LM} = u(b)$. This is only the case when the employment contract delivers $\lambda(u(wl) - l) = u(b)$ to the employees. Job applicants are thus indifferent between unemployment and finding a job. However, such an employment contract is only possible when $l = q_s$ and $w = p$, so that the difference between payroll- and self-employment disappears. Moreover, as $\lambda\psi(p) - u(b)$ also features in the free-entry condition, workers' indifference is only consistent with vacancy creation when $k = 0$. For a given b , firms then must open exactly so many vacancies such that aggregate supply results in a price such that the condition $\lambda\psi(p) = u(b)$ is satisfied. ■

A similar insight can be derived in the baseline model with large firms, under parameter values that render the insurance offered by firms irrelevant. To see this, consider the case with $u(b) = 0$ for simplicity, so that equilibrium is described by conditions (11) - (12). Now suppose that the frictions in the goods market completely vanish: $\lambda \mapsto 1$. Then, an MSCC equilibrium can only exist when employees capture the full surplus from the match, $\phi \mapsto 1$, and search frictions in the labour market vanish as well, $\mu(\theta) \mapsto 1$. However, a job-finding probability tending to one is only compatible with costless job creation by the virtue of (9), so that k must tend to zero. Furthermore, (10) implies that $l = q_s$ in the limit, and thus $w = p$. Consequently, when $\lambda \mapsto 1$, payroll- and self-employment become exactly identical so that the type-of-employment choice margin is ill-defined again.

Therefore, payroll employment can be interpreted as a costly insurance mechanism that is delivered by firms against goods market risk. If such insurance either cannot be provided or is irrelevant, payroll- and self-employment become indistinguishable, and the creation of payroll employment must happen at no cost. Otherwise, the only equilibrium is a corner case of $SE = 1$.

3.2.2 Consequences of ignoring type-of-employment choice

Theorem 2 shows that our model overturns the standard result on the effect of matching efficiency on the job-finding rate. To highlight that allowing workers to choose their type of

employment is the reason for this implication of the model, we define an equilibrium without type-of-employment choice, in which the self-employment rate is exogenously fixed. Such an equilibrium can be rationalised, for example, by assuming that selection into self-employment is purely based on individual-specific preferences or costs, regardless of the income differential between the two types of employment.

Definition 3 (Fixed career-choice equilibrium) *A fixed career-choice equilibrium (FCC-equilibrium) is a tuple $(p, q_s, q_c, w, l, \theta)$ such that, given $SE \in (0, 1)$:*

- *given p , each consumer demands q_c as prescribed by equation (1), each self-employed sells q_s given by equation (2) and θ, w, l satisfy equations (3) - (5) given p ,*
- *price p clears the goods market so that equation (6) holds.*

When we fix the split of workers between self-employment and pursuing a career at a firm, the predictions on how reductions in search frictions in the goods and the labour market affect the job-finding rate change their signs, as summarised in the following result.

Proposition 2 (Comparative statics of reductions in search frictions in FCC-equilibrium)

For any FCC-equilibrium, the job-finding probability decreases in the ease of entry in the goods market and increases in the matching efficiency in the labour market, $d\mu(\theta)/d\lambda < 0$ and $d\mu(\theta)/dE > 0$.

The proof is provided in Appendix A.5. Ignoring type-of-employment choice may therefore lead to incorrect inference on the effect of improvements in labour market matching efficiency on unemployment. This prediction changes sign because the inflow of workers from self-employment, which (more than) offsets the initial increase in the job-finding probability in an MSCC-equilibrium, is ruled out in an FCC-equilibrium. The effect of λ on the job-finding probability also switches sign, for two reasons. First, improvements in goods market frictions act as a positive supply shock that decreases the price level, depressing job creation by large firms. Second, the resulting decrease in labour market tightness is not mitigated by job applicants leaving the labour market for self-employment. This adjustment in the composition of employment, in the baseline model, led to a price increase, and this mechanism is absent in an FCC-equilibrium.

3.3 Extensions

Our parsimonious framework focuses on risk differentials and hence abstracts from other aspects of the type-of-employment choice. In this section we show how the model can be extended with three of them. First, we address the difference in permanency between payroll- and self-employment. Second, we introduce inactive workers in the model, making participation in the labour force a costly choice. Third, we allow for individual preferences for, or costs of, self-employment.

3.3.1 Payroll employment as a long-term relationship

The static model cannot capture the long-term nature of many payroll employment relationships. To include long-term employment relationships, we build a dynamic version of the model. In

this model, agents discount future utilities at a rate of β and employee-vacancy matches are destroyed at the end of each period with probability δ . Production cannot be stored. Here we discuss how this dynamic extension affects the existence of an MSCC equilibrium, while the details and generalisations of our earlier results can be found in Appendix C.4. The dynamic counterparts to equilibrium conditions (11) and (12) are:

$$\lambda = \frac{\mu(\theta)\phi}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)},$$

$$k = \frac{\zeta(\theta)(1 - \phi)}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)} p\psi(p).$$

The conditions of the static model are a special case of these equations when all payroll jobs are destroyed at the end of the first period, $\delta = 1$. The steady-state payroll employment PE is:

$$PE = \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} (1 - SE),$$

so that the goods market clears for:

$$\lambda \left(SEq_s + (1 - SE) \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l \right) = q_c.$$

Qualitatively, this model works exactly as the baseline case and the comparative statics are identical. Moving away from the special case of $\delta = 1$ and keeping the same λ requires a lower job finding rate $\mu(\theta)$ to support an MSCC-equilibrium in the dynamic model. This is because the long-term nature of payroll employment is an additional benefit to applicants. To restore indifference between the two careers, the self-employment rate must be smaller than in the static model.

3.3.2 Participation margin

In the baseline model we focus on active members of the labour force. We can extend the model with inactive workers by postulating a fixed cost of labour force participation. Without loss of generality, let this cost be distributed according to $\text{Unif}_{[0,1]} \times F(j)$, on the unit square inhabited by workers, where $F(j)$ is a continuous cdf.¹³ Let workers indexed with $j \in [0, 1]$ pay a cost $c_a(j)$ upon entering the labour force, with $c'_a(j) \geq 0$. This cost cancels out in the type-of-employment choice and is sunk at the contract-posting stage in the labour market. Focusing on the more tractable case of $u(b) = 0$, the adjustments to the equilibrium conditions are:

$$A = (1 - SE)F(j),$$

$$F(j)\lambda (SEq_s + (1 - SE)\mu(\theta)l) = q_c,$$

$$\lambda\psi(p) = \mu(\theta)\phi\psi(p) \geq c_a(j).$$

¹³ This permits us to consider a threshold, instead of a continuum of thresholds, for the participation decision of workers.

The first equation simply states that SE is the share of *active* workers who choose self-employment. The second equation is the goods-market clearing condition adjusted for participation. Finally, the values of entering the labour market or self-employment, V^{LM} and V^{SE} , either pin down the equilibrium index $j^* < 1$ that solves the three equations with a binding equality, or exceed $c(j)$ so that $j^* = 1$ and $F(j^*) = 1$ and even the highest participation cost is covered in equilibrium.

Theorem 2 is robust to this extension, as the effects of E and λ on p and θ are not affected by the participation margin. Consider, for example, an increase in λ . The differentiation of the free-entry condition and workers' indifference goes through exactly as in the baseline model. Consequently, it is still the case that the equilibrium price p must increase when λ increases. Therefore, declining goods market frictions increase the value of self-employment V^{SE} and also lead to higher participation as long as $j^* < 1$. Higher participation exerts a downward pressure on the equilibrium price, so that the eventual increase in p comes from the newly active workers selecting disproportionately more into self-employment.

3.3.3 Preference for self-employment

Last, but not least, the model can be extended with a cost of or premium to self-employment, by making distributional assumptions analogous to costs of labour force participation. In particular, assume that workers indexed with $j \in [0, 1]$ pay a cost $c_{se}(j)$ if they enter self-employment, with $c'_{se}(j) \geq 0$. This general specification can capture, e.g., preferences for being one's own boss (Hurst and Pugsley, 2011), or financial frictions (Buera, 2009, Evans and Jovanovic, 1989) in reduced form. Negative values of $c_{se}(j)$ then correspond to individual preferences for or subsidies to self-employment. For a continuous distribution of $c_{se}(j)$ with sufficiently broad support, the marginal workers j^* are pinned down by the indifference condition:

$$\lambda\psi(p) - c_{se}(j^*) = \phi\mu(\theta)\psi(p),$$

and $SE = F(j^*)$. The shifter $c_{se}(j)$ is irrelevant for the contract-posting problem, and also the free-entry condition is the same as in the baseline model. Then, any equilibrium of the baseline model can be regarded as an equilibrium of this economy with the distribution of c_{se} such that the equilibrium index j^* has $c_{se}(j^*) = 0$.

In the case of a continuous distribution of c_{se} , the preference for self-employment tames the responses of the equilibrium self-employment rate to λ and E . The next type in the distribution of $c_{se}(j)$ is less inclined to take up self-employment than the j^* -th type, so that the effects of shocks that increase the self-employment rate in the baseline model are now muted. The same goes for shocks that would decrease the self-employment rate, because the affected workers were more inclined than the marginal worker to select into self-employment.

4 Empirical Results

The theoretical model yields sharp predictions for the response of the aggregate self-employment rate to the parameters that capture the magnitude of search frictions in the goods and in the

labour market. However, in practice it is hard to distinguish reductions in labour market frictions from reductions in goods market frictions. Instead, we study the effect of the most salient recent reduction in market frictions, the roll-out of broadband Internet.

High-speed Internet has been accompanied by the development and use of platforms and other apps that facilitate trade. We think of these innovations as increasing the likelihood that a seller can be found by a customer. Because broadband Internet penetration measures the extent to which customers and the self-employed have access to these technologies, we use it as proxy for λ . Internet also facilitates conventional job search and recruiting (Autor, 2001, Stevenson, 2009). It has reduced both the costs of job applications and the costs of screening each application. Although the interaction of these cost reductions is not unambiguous (see e.g. Albrecht et al. (2006)), there is some evidence of increases in matching efficiency (Martellini and Menzio, 2020). We therefore use broadband penetration as a proxy for E as well.

We test whether broadband Internet penetration, which we expect to reduce search frictions, correlates with self-employment rates or not. If there is a significant effect of broadband Internet on self-employment, we can use its sign to learn whether the goods market or the labour market channel dominates. The former would correspond to a positive effect of Internet on the self-employment rate while the latter to a negative effect. Because we explicitly aim to study general equilibrium effects, we need to test the strength of our channels at the level of the entire market. Thus, we rely on longitudinal cross-country variation in self-employment rates and broadband Internet.

Switching careers may take time. To allow the effects of the reductions in frictions to be distributed over time, we follow the literature (Parker, 2004, Robson and Wren, 1999) and specify a generalized error-correction model:

$$\Delta \ln SE_{it} = \beta \Delta X_{it} + \omega X_{it-1} - \gamma \ln SE_{it} + \alpha_i + \psi_t + \varepsilon_{it}. \quad (20)$$

In this equation, the dependent variable is the difference of (the logarithm of) the self-employment rate. On the right-hand side we have country and time fixed-effect variables α_i and ψ_t , respectively. These control for country-specific invariant characteristics, e.g. cultural attitudes to entrepreneurship, and for common time variation, e.g. the global business cycle. X_{it} denotes a list of regressors which we describe below. Following the logic of the error-correction model, the β coefficients estimate the short-run effects of X_{it} on the dependent variable, while the ratios $\omega/|\gamma|$ measure the long-run effects.

4.1 Data

Self-employment rates For the composition of employment we sample ILOSTAT data. Even though, in the model, all self-employed face the same amount of income risk, in reality, they are a heterogeneous group in this regard. On the one hand, it is fair to argue that members of producers' cooperatives enjoy some insurance provided by the cooperative. An employer running a large incorporated business with many employees does not fit the definition of the self-employed in the model either. However, focusing solely on own-account workers omits unincorporated employers with very few employees who have similar characteristics to

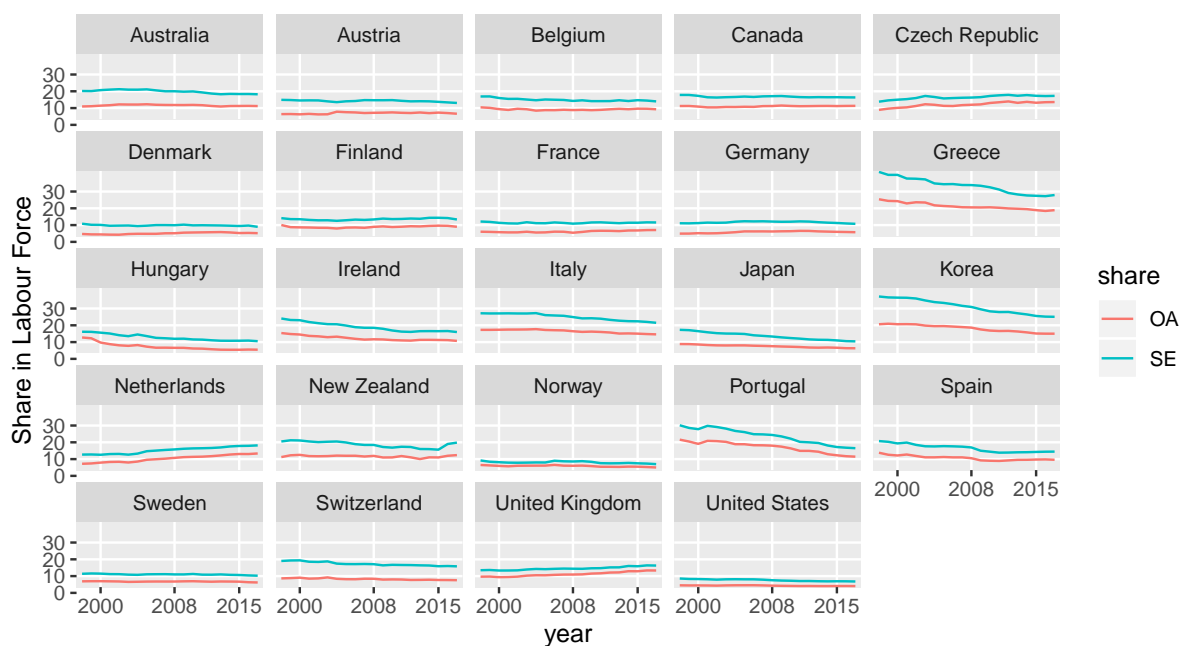


Figure 4: Self-employment (SE) and own-account work (OA) shares in labour force (in percentages) for 24 OECD countries between 1998 and 2017.

own-account workers, as shown by Levine and Rubinstein (2016).

To address these concerns, we use two proxies for the SE_{it} variable in equation (20): self-employment and own-account work. We define the *Self-Employment* rate as the share of the self-employed in the labour force and the *Own-Account Work* rate as the share of own-account workers in the labour force. Figure 4 presents the evolution of these variables between 1998 and 2017 for the 24 OECD countries for which we have information on broadband Internet as well. We see that self-employment is either flat or decreasing, except for the Czech Republic, Netherlands, and the United Kingdom.

Broadband The broadband penetration rate is defined as the number of broadband subscribers per 100 inhabitants.¹⁴ We take these data from the ITU World Telecommunication/ICT Indicators Database, in which broadband Internet appears in 1998 (in seven countries only, with each less than 0.5% penetration). To control for potential endogeneity, which could arise if an increase in gig-economy self-employment would prompt further investment in broadband technology, we follow the instrumental-variable approach introduced in Czernich et al. (2011). The idea behind this instrument is that the most commonly used broadband standards use pre-existing infrastructure to connect homes and small- and medium-sized firms to the larger network. In particular, the copper wire of the voice telephony network and the coaxial cable of the cable TV network are employed to connect individual users to the Internet. Since the voice telephony and cable TV network have been built for other purposes than broadband Internet, they provide valid instruments for broadband penetration.

Instrument relevance has been shown by Czernich et al. (2011). They find that the adoption

¹⁴ Broadband Internet offers download speeds of at least 256 kbit/s. One subscription usually covers the whole household/establishment.

Table 1: Effects of Broadband Internet on Self-Employment Rate

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate
Lagged SE rate	-0.074*** (0.020)	-0.074*** (0.021)	-0.100*** (0.028)	-0.185*** (0.032)	-0.090*** (0.020)	-0.176*** (0.036)
Δ Predicted B-band	0.024*** (0.008)	0.025*** (0.008)	0.020** (0.009)	0.042*** (0.009)	0.029*** (0.009)	0.038*** (0.012)
Lagged Predicted B-band	0.003*** (0.001)	0.003*** (0.001)	0.002** (0.001)	0.006*** (0.001)	0.004*** (0.001)	0.006*** (0.002)
Δ GDP		0.008 (0.073)	0.012 (0.080)	0.010 (0.076)	0.010 (0.082)	0.034 (0.080)
Lagged GDP		0.011 (0.028)	0.025 (0.030)	-0.009 (0.032)	0.037 (0.029)	-0.023 (0.038)
Δ Replacement Rate			0.000 (0.011)			0.038 (0.031)
Lagged Replacement Rate			-0.010 (0.006)			0.008 (0.020)
Δ Public Sector				-0.066* (0.035)		-0.015 (0.045)
Lagged Public Sector				-0.104*** (0.027)		-0.105*** (0.030)
Δ Tax Burden					0.036 (0.038)	0.058 (0.039)
Lagged Tax Burden					-0.012 (0.020)	-0.003 (0.022)
Observations	456	456	384	249	408	211
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
F-test p -value	0.000	0.000	0.000	0.000	0.001	0.009
AR(1) coefficient	0.117	0.118	0.124	0.112	0.133	0.227
Modified Wald p -value	0.000	0.000	0.000	0.000	0.000	0.000
RSS	0.305	0.305	0.253	0.101	0.274	0.081
MSS	0.038	0.038	0.033	0.036	0.039	0.024
RMSE	0.027	0.027	0.026	0.022	0.028	0.022
R-squared	0.110	0.111	0.115	0.261	0.125	0.231

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the *Self-Employment* rate. The sample is 24 OECD countries in years 1998-2017. Robust standard errors in parentheses.

of broadband Internet is well-described by a logistic diffusion curve, where the pre-existing voice telephony and cable TV infrastructure places a bound on the maximum reach of the broadband network in a country. Broadband penetration for each country and year can then be predicted with a nonlinear regression, featuring the maximum reach, the speed, and the inflection point of the diffusion process as parameters. Moreover, Czernich et al. (2011) show that the pre-existing infrastructure of the cable TV and voice telephony networks has no predictive value for the penetration of mobile telephony and computers, two other technologies that have been widely adopted around the same time as broadband Internet. Regarding the first-stage estimation, we merely extend their work to a longer time period. We report the details of this exercise, which results in our regressor *Predicted Broadband*, in Appendix B.3.

Additional regressors At the benefit of its analytical tractability, our parsimonious model necessarily abstracts from several variables that the literature has found to impact cross-country

Table 2: Effects of Broadband Internet on Own-Account Work

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate
Lagged OA rate	-0.112*** (0.027)	-0.110*** (0.026)	-0.112*** (0.033)	-0.186*** (0.032)	-0.107*** (0.029)	-0.213*** (0.041)
Δ Predicted B-band	0.017* (0.009)	0.018* (0.008)	0.020* (0.011)	0.050*** (0.012)	0.029*** (0.007)	0.050*** (0.016)
Lagged Predicted B-band	0.002* (0.001)	0.002* (0.001)	0.003* (0.002)	0.007*** (0.002)	0.004*** (0.001)	0.008*** (0.002)
Δ GDP		-0.158 (0.106)	-0.180 (0.114)	-0.127 (0.099)	-0.179 (0.111)	-0.116 (0.104)
Lagged GDP		-0.002 (0.048)	-0.000 (0.039)	-0.055 (0.043)	0.024 (0.045)	-0.066 (0.050)
Δ Replacement Rate			-0.003 (0.011)			0.029 (0.041)
Lagged Replacement Rate			-0.008 (0.010)			0.000 (0.027)
Δ Public Sector				-0.071 (0.046)		-0.030 (0.059)
Lagged Public Sector				-0.102*** (0.034)		-0.127*** (0.039)
Δ Tax Burden					-0.020 (0.027)	-0.007 (0.052)
Lagged Tax Burden					-0.013 (0.022)	-0.001 (0.029)
Observations	456	456	384	249	408	211
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
F-test p-value	0.000	0.000	0.000	0.000	0.000	0.026
AR(1) coefficient	0.072	0.071	0.0577	0.127	0.058	0.193
Modified Wald p-value	0.000	0.000	0.000	0.000	0.000	0.000
RSS	0.553	0.550	0.428	0.175	0.467	0.139
MSS	0.078	0.081	0.050	0.055	0.059	0.038
RMSE	0.036	0.036	0.034	0.029	0.035	0.029
R-squared	0.123	0.128	0.104	0.239	0.112	0.213

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the *Own Account Work* rate. The sample is 24 OECD countries in years 1998-2017. Robust standard errors in parentheses.

self-employment rates. Because omitting such factors would likely bias our results, we incorporate four additional regressors in our analysis. First, we control for GDP per capita, *GDP*. The earlier literature finds a negative relationship between the general level of economic development, proxied with GDP per capita, and the self-employment rate (Acs et al., 1994, Poschke, 2019).

Second, we account for the generosity of the official unemployment insurance system. In particular, following Parker and Robson (2004), Staber and Bögenhold (1993), Torrini (2005) and Kumar (2012), we control for the *Replacement Rate*, defined as the ratio of the unemployment benefit to the median wage. The more generous the UI system, the more likely prospective workers are to engage in job search rather than self-employment. This can arise due to moral hazard problems which make monitoring search effort difficult.¹⁵

¹⁵ In Appendix C.1 we derive analytically, under parametric assumptions, that the introduction of UI adversely affects the equilibrium self-employment rate.

Third, following Parker and Robson (2004), Robson and Wren (1999) and Torrini (2005), the choice to become a self-employed can also be driven by tax evasion. Therefore, the higher taxes are, the larger the self-employment rate is likely to be. We thus define the *Tax Burden* as the average of the tax wedge for a single-earner household, and married couples with two children and a single earner.

Finally, we introduce the *Public Sector* variable as the share of public-sector employment in total employment, which is shown to be relevant for the aggregate self-employment rates in Torrini (2005). This variable proxies for the incidence of safe, stable jobs in the economy. Therefore, we expect that countries with big public-sector employment have lower self-employment rates. The justification for this hypothesis is that even though firms in our model offer perfect insurance, in reality there are certain impediments for them to do so (e.g. moral hazard) and government jobs come with stability. Further details of how the variables were constructed can be found in Appendix B.1. We report descriptive statistics of our sample in Table 3 in Appendix B.2.

4.2 Results

We investigate equation (20) in several steps, varying the contents of the X_{it} matrix. We always employ *Predicted Broadband* in our regressions. Then, we add *GDP* and next incorporate the institutional variables one by one and finally, we estimate the model with all regressors.¹⁶ Since the data on institutional variables provided by the OECD usually start two to three years later than our data on broadband, there is trade-off between the length of the sample and the number of regressors we use. We report the results of our estimation in Tables 1 for *Self-Employment* and 2 for *Own Account Work* as the dependent variables. Several insights emerge.

First, the signs of the coefficients on differenced and lagged *Predicted Broadband* are positive. This is in line with our model predictions when the reduction of goods market frictions is the dominant effect of Internet. We find evidence of both short- and long-term effects. Second, we find evidence for a negative correlation between the importance of public-sector employment and the dependent variables. This finding further corroborates the main premise of our model that selection into self-employment is affected by income insurance that payroll jobs provide.

For both dependent variables, the *F*-test rejects the null hypothesis of all regression coefficients being equal to zero. We do not find convincing evidence in favor of serial correlation of the residuals, which we test for by estimating the regressions while we explicitly allow for an AR(1)-process in the residuals. However, the modified Wald test rejects the null hypothesis of no groupwise heteroscedasticity. Thus, because of cross-sectional heteroscedasticity, we report robust standard errors.

As a robustness check, we estimate a first-differenced (FD) version of equation (20) that relies on weaker assumptions on the error term. This procedure removes the unobserved fixed effects α_i at the cost of shrinking the length of the sample. We report the result of this exercise together with the estimation results of the specification that allows for an AR(1)-process in the residuals in Appendix B.4.

¹⁶ All regressors, apart from *Predicted Broadband* enter the estimation in logarithms and differences of logarithms. This is due to zero and near-zero observations for *Predicted Broadband*.

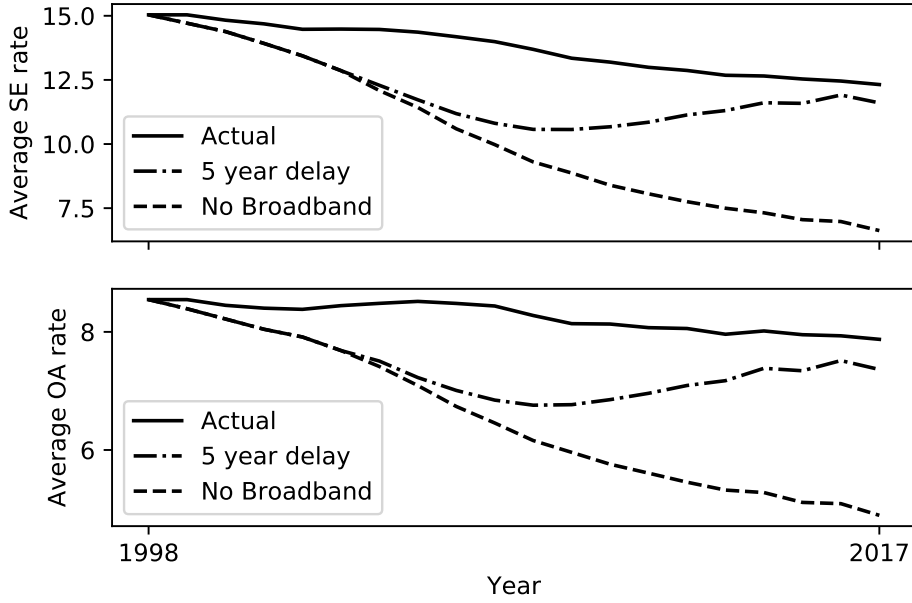


Figure 5: Counterfactual predictions of either lack of, or delay of, broadband development for self-employment (top) and own account work rates (bottom) in a labour-force-weighted sample of countries.

The dynamic and auto-regressive formulation of the estimation equation (20) makes the inference on the quantitative significance of the estimated coefficients difficult. Thus, we consider a hypothetical stark scenario of no broadband Internet, setting *Predicted Broadband* to zero in all of the sample. Then, we calculate predicted *Self-Employment* and *Own-Account Work* rates using estimates from column (2) of Tables 1 and 2.¹⁷ Then, we calculate counterfactual weighted-average shares of self-employed and own-account workers in the labour force in each year, with weights for each country given by the size of the labour force of a country in a given year.

The results of these exercises are illustrated on Figure 5. We uncover a strong accumulative effect of fast Internet diffusion on self-employment. If it was not for broadband Internet, the drop in the *Own Account Work* rate from the initial value of 8.6% would be not to 7.9% but to 4.1% instead. The effect on *Self-Employment* rate is even stronger. The initial average of 15% would drop not to 12.3% but to 4.6% in 2017.

For a more complete picture, we also consider a counterfactual of a delayed broadband Internet roll-out, assuming it was introduced not in 1998, but in 2003 instead, forwarding the observations of *Predicted Broadband* by five years. If, instead, the broadband Internet roll-out had not been completely absent, but merely delayed by 5 years, the average *Self-Employment* and *Own-Account Work* rates would have been reverting towards the values observed in the data after an initial decline. The difference between the counterfactual and the observed average in

¹⁷ We chose this specification as the one with the largest number of regressors that still allows us to use our full sample. For example, the data on *Replacement Rate* only start in 2001. Thus, using a richer specification would not only shrink the size of the sample significantly, but it would also yield baseline observations of the dependent variables that already include the effects of at least 4 years of broadband Internet roll-out.

2017 for *Self-Employment* is approximately 0.7 percentage point and for *Own-Account Work* it is about 0.5 percentage point. This experiment essentially removes the last five years of broadband Internet roll-out from our sample, while the most dynamic broadband diffusion happened between 2000 and 2005. This period is responsible for the bulk of the effect of broadband on self-employment.

5 Conclusions

We introduce goods market frictions in a stylised model of type-of-employment choice and apply it to understand the effects of improvements in ICT technologies on the self-employment rate. In our model, the composition of employment is driven by income risk and firms' ability to insure their employees against it. We show that a reduction in goods market frictions increases self-employment, even though firms benefit from such a reduction as well, because it decreases the value of insurance that firms offer.

We use our theory to disentangle the goods market and labour market effects of improvements in ICT technologies. Using cross-country data on the self-employment rate and pre-existing infrastructure as instrument for broadband penetration, we find that the results confirm the origin story of the gig economy that fast Internet prompts more self-employment. Without the introduction of broadband Internet, the self-employment rate in our sample would fall by about three-quarters.

Our model allows us to infer that the general equilibrium effects of overall reductions in market frictions on self-employment, spurred by broadband Internet penetration, were due to the goods market channel dominating the labour market channel. Moreover, allowing for type-of-employment choice overturns the standard result that increases in labour market matching efficiency improve the job-finding probability, and therefore provide an alternative explanation for why improvements in labour market matching efficiency do not result in lower unemployment rates.

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A Proofs and Derivations

A.1 General contract posting problem

The FOCs for the problem read, with χ being the multiplier on the market utility constraint:

$$\frac{\partial \Pi}{\partial \theta} : \quad \zeta'(\theta) (\lambda(p - w)l) = \chi \mu'(\theta) (\lambda (u(wl) - l) + (1 - \lambda)u(d) - u(b)) \quad (21)$$

$$\frac{\partial \Pi}{\partial l} : \quad \zeta(\theta)\lambda(p - w) = \chi \mu(\theta)\lambda (u'(wl)w - 1) \quad (22)$$

$$\frac{\partial \Pi}{\partial w} : \quad -\zeta(\theta)\lambda l = \chi \mu(\theta)\lambda u'(wl)l \quad (23)$$

$$\frac{\partial \Pi}{\partial d} : \quad -\zeta(\theta)(1 - \lambda) = \chi \mu(\theta)(1 - \lambda)u'(d) \quad (24)$$

Let's have a closer look at equations (23) and (24). After canceling some terms they can be combined to yield:

$$u'(d) = u'(wl) \implies d = wl. \quad (25)$$

Hence, all employees receive identical consumption, but only those able to work exert effort l . d as a control is redundant as it's fully determined by optimal choices of w and l . Factoring in equation (25) we can simplify the remaining FOCs. Merging (22) and (23) we arrive at:

$$u'(wl) = \frac{1}{p}.$$

Combining (22) and (21) yields:

$$\phi(\lambda p - w)l = (1 - \phi) \frac{(u(wl) - \lambda l - u(b))}{u'(wl)}.$$

A.2 Proof of Lemma 1

Proof.

1. Here we show the results regarding the characteristics of the labour income distribution, namely: $wl = pq_s$, $w < p$ and $q_s < l$. Combining equations (2) and (3) we have:

$$u'(pq_s) = \frac{1}{p} = u'(wl) \implies wl = pq_s.$$

Next, for firms to make positive profits per filled vacancy it must hold that $w < \lambda p$ so that $w < p$, and thus $q_s < l$ follows trivially.

Finally, we will see below that for $u(b) = 0$, $\mu(\theta) > \lambda$. However, positive profits per filled vacancy require that $\lambda p > w$. Hence $\mu(\theta)p > w$, which implies $\mu(\theta)l > q_s$ since $pq_s = wl$.

2. To obtain function $\psi(p)$, we start with removing q_s in V^{SE} . From (2):

$$q_s = \frac{1}{p} [u']^{-1} \left(\frac{1}{p} \right)$$

with $[u']^{-1}(\cdot)$ being the inverse of marginal utility. Therefore, we have:

$$V^{SE} = \lambda(u(pq_s) - q_s) = \lambda \left(u \left([u']^{-1} \left(\frac{1}{p} \right) \right) - \frac{1}{p} [u']^{-1} \left(\frac{1}{p} \right) \right),$$

so that:

$$\psi(p) \equiv u \left([u']^{-1} \left(\frac{1}{p} \right) \right) - \frac{1}{p} [u']^{-1} \left(\frac{1}{p} \right) = u(pq_s) - q_s,$$

and $V^{SE} = \lambda\psi(p)$. To derive the sharing rule, we substitute (3) into (4) to arrive at:

$$\begin{aligned} \phi(\lambda p - w)l &= (1 - \phi) p (u(wl) - \lambda l - u(b)) \implies \\ \lambda l &= (1 - \phi) \left(u(wl) - \frac{wl}{p} - u(b) \right) + \frac{wl}{p}, \end{aligned} \quad (26)$$

which equals (10) since $wl = pq_s$, so that $u(wl) - wl/p = u(pq_s) - q_s = \psi(p)$. We shall now demonstrate that $\psi(p)$ shows up in V^{LM} too. Incorporating (26), the value of pursuing a career at a firm reads:

$$V^{LM} = \mu(\theta) \left(u(wl) - (1 - \phi) \left(u(wl) - \frac{wl}{p} - u(b) \right) - \frac{wl}{p} \right) + (1 - \mu(\theta)) u(b),$$

so that:

$$V^{LM} = \mu(\theta) \phi \left(u(wl) - \frac{wl}{p} \right) + u(b) - \mu(\theta) \phi u(b).$$

Therefore, we can write:

$$V^{LM} = u(b) + \mu(\theta) \phi [\psi(p) - u(b)].$$

This equation, together with the definition of V^{SE} , yields the workers' indifference condition in (8). For $u(b) = 0$, this condition simplifies to

$$\lambda = \phi \mu(\theta),$$

so that $\lambda < \mu(\theta)$ since $\phi < 1$.

To derive the free-entry condition in (9), substitute (26) into (5) with $wl = pq_s$, so that:

$$\begin{aligned} k &= \zeta(\theta) p ((1 - \phi) (u(pq_s) - u(b)) + \phi q_s - q_s), \\ &= \zeta(\theta) (1 - \phi) p [\psi(p) - u(b)]. \end{aligned}$$

Finally, we show that $\psi(p)$ is increasing in p . Since $\psi(p) = u(pq_s) - q_s$, we have

$$\psi'(p) = u'(pq_s) q_s + \frac{dq_s}{dp} (pu'(pq_s) - 1).$$

However, in the optimum the second term is equal to zero, so that $\psi'(p) = u'(pq_s) q_s > 0$.

3. To show that the demand function is strictly monotone in p , differentiate condition (1):

$$g''(pq_c) \left(q_c + p \frac{dq_c}{dp} \right) = -\frac{1}{p^2} \implies \frac{dq_c}{dp} = \frac{-\frac{1}{p^2} \frac{1}{g''(pq_c)} - q}{p} < 0.$$

■

A.3 Proof of Theorem 2

Proof. The proof relies on the three equilibrium conditions (the free entry condition, workers' indifference condition, and goods market clearing condition) and the relationships they imply between p, θ and SE . Furthermore, we first derive the results for the special case of $b = 0$. The signs of the derivatives there will help us sign the effects of model parameters in the more general case of $b > 0$. Observe that the MSCC-equilibrium is a level set of a mapping described

by the three remaining equilibrium conditions involving the utility function u , matching probabilities $\mu(\theta)$ and $\zeta(\theta)$, and $\psi(p)$ (which inherits smoothness from the utility function). Thus, any element of an MSCC is at least once continuously differentiable.

The case of $b = 0$. Defining $\gamma(p) = p\psi(p)$ for brevity, the workers' indifference and free-entry conditions in (11) and (12) become:

$$\begin{aligned}\lambda &= \mu(\theta)\phi, \\ k &= \zeta(\theta)(1 - \phi)\gamma(p).\end{aligned}$$

- a. **Effects of λ on $\mu(\theta)$ and θ .** Implicitly differentiating this simplified career-choice indifference condition we arrive at:

$$1 = \underbrace{[\mu'(\theta)\phi]}_{>0} \frac{d\theta}{d\lambda}.$$

Trivially, this amounts to $d\mu(\theta)/d\lambda = 1 > 0$, but it also reveals that $d\theta/d\lambda > 0$.

- b. **Effect of λ on p .** Next, we differentiate the free-entry condition:

$$0 = \underbrace{[\zeta'(\theta)(1 - \phi)] \frac{d\theta}{d\lambda} \gamma(p)}_{<0} + \underbrace{\zeta(\theta)(1 - \phi) \gamma'(p)}_{>0} \frac{dp}{d\lambda}.$$

By standard properties $\zeta'(\theta) < 0$, so that the first term is negative. By the virtue of Lemma 1, as $\gamma(p) = p\psi(p)$, $\gamma'(p) > 0$. Hence, this equality can only hold if $dp/d\lambda > 0$.

- c. **Effect of λ on SE .** Finally, consider the goods-market clearing condition in (6). We can differentiate it with respect to λ :

$$\underbrace{SEq_s + (1 - SE)\mu(\theta)l}_{>0} + \lambda \left(\underbrace{\frac{dSE}{d\lambda} (q_s - \mu(\theta)l)}_{>0} + \underbrace{SE \frac{dq_s}{dp} \frac{dp}{d\lambda}}_{>0} + \underbrace{(1 - SE) \left(\mu'(\theta) \frac{d\theta}{d\lambda} l + \mu(\theta) \frac{dl}{dp} \frac{dp}{d\lambda} \right)}_{>0} \right) = \underbrace{\frac{dq_c}{dp} \frac{dp}{d\lambda}}_{<0}.$$

By Assumption 1, the quantity supplied by the self-employed q_s increases in p and the converse is true for the quantity demanded q_c , as stated in Lemma 1. We can also show that $dl/dp > 0$ as from (10):

$$\begin{aligned}\lambda l &= (1 - \phi) [\psi(p) - u(b)] + q_s \implies \\ \lambda \frac{dl}{dp} &= (1 - \phi) \psi'(p) + \frac{dq_s}{dp} > 0.\end{aligned}\tag{27}$$

Hence, the differentiation of the goods market clearing condition can be summarised as

the following sign restriction:

$$\frac{dSE}{d\lambda} (q_s - \mu(\theta)l) < 0. \quad (28)$$

By virtue of Lemma 1 we know that $\mu(\theta)l > q_s$, so that $dSE/d\lambda > 0$.

- d. **Effects of E on $\mu(\theta)$ and θ .** Now, let us consider improvements in matching efficiency in the labour market. We have that $\mu(\theta) = \lambda/\phi$ and is constant. Hence, E cannot have any effect of $\mu(\theta)$ when $b = 0$. Moreover, since E increases while $\mu(\theta) = E\hat{\mu}(\theta)$ is constant, we find that $d\theta/dE < 0$. As a result the job-filling rate $\zeta(\theta) = E\hat{\zeta}(\theta)$ goes up.
- e. **Effects of E on p and SE .** Given that $d\theta/dE < 0$, it follows that the derivative of p with respect to E has its sign opposite to that of its counterpart derivative with respect to λ . To see that this derivative switches sign, differentiate the free-entry condition to obtain

$$0 = \underbrace{\hat{\zeta}(\theta) (1 - \phi) \gamma(p)}_{>0} + \underbrace{\zeta'(\theta) (1 - \phi) \gamma(p) \frac{d\theta}{dE}}_{>0} + \underbrace{\zeta(\theta) (1 - \phi) \gamma'(p)}_{>0} \frac{dp}{dE}.$$

To see that the derivative of SE with respect to E switches sign too compared to its counterpart, note that differentiating the goods-market clearing condition in (6) with respect to E yields:

$$\lambda \left(\frac{dSE}{dE} (q_s - \mu(\theta)l) + \underbrace{SE \frac{dq_s}{dp} \frac{dp}{dE}}_{<0} + (1 - SE) \left(\underbrace{\frac{d\mu(\theta)}{dE} l}_{=0} + \underbrace{\mu(\theta) \frac{dl}{dp} \frac{dp}{dE}}_{<0} \right) \right) = \underbrace{\frac{dq_c}{dp} \frac{dp}{dE}}_{>0},$$

so that $dSE/dE < 0$. Because of free entry of vacancies while $\zeta(\theta)$ increases, the price p must fall. Prices fall because SE goes down, resulting in an inflow in the labour market keeping $\mu(\theta)$ unchanged.

The case of $b > 0$.

- a. **Effect of λ on p .** Implicitly differentiating the workers' indifference and free-entry conditions in (8) and (9), we arrive at:

$$\psi(p) + \lambda \psi'(p) \frac{dp}{d\lambda} = \mu'(\theta) \frac{d\theta}{d\lambda} \phi [\psi(p) - u(b)] + \mu(\theta) \phi \psi'(p) \frac{dp}{d\lambda}, \quad (29)$$

$$0 = \zeta'(\theta) \frac{d\theta}{d\lambda} p [\psi(p) - u(b)] + \zeta(\theta) (\psi(p) - u(b) + p \psi'(p)) \frac{dp}{d\lambda}. \quad (30)$$

Suppose that $dp/d\lambda = 0$. Then, (30) requires $d\theta/d\lambda = 0$ while (29) can only hold when $d\theta/d\lambda > 0$. Hence, by an argument of continuous differentiability, and using that $dp/d\lambda > 0$ at $u(b) = 0$, we can rule out $dp/d\lambda \leq 0$. Thus, $dp/d\lambda$ will not change its sign when we increase b as long as we remain in an MSCC-equilibrium.

- b. **Effect of λ on θ .** Having established that $dp/d\lambda > 0$ in an MSCC-equilibrium, we can turn our attention to $d\theta/d\lambda$. Note in (8) that for $b = 0$ we have $\lambda = \mu(\theta)\phi$, while for the

mixed-strategy equilibrium to exist for $b > 0$, we need $\lambda > \mu(\theta)\phi$. Rearranging (29) we have:

$$\psi(p) + \underbrace{(\lambda - \mu(\theta)\phi)}_{>0} \psi'(p) \frac{dp}{d\lambda} = \mu'(\theta) \frac{d\theta}{d\lambda} \phi (\psi(p) - u(b)),$$

which implies $d\theta/d\lambda > 0$ since in any MSCC-equilibrium $\psi(p) > u(b)$.

- c. **Effect of λ on SE .** Note in (27) that $dl/dp > 0$ does not depend on b being zero, so that the sign restriction in (28) still holds. We know that at $b = 0$ we have $dSE/d\lambda > 0$ and $q_s - \mu(\theta)l < 0$. Since their product must be negative, neither of those can be equal to 0 for $b > 0$. From continuity we thus get that the same comparative statics hold for $b > 0$: the self-employment rate SE increases in λ .
- d. **Effects of E .** Again, we write $\mu(\theta) = E\hat{\mu}(\theta)$ and $\zeta = E\hat{\zeta}(\theta)$, so that the workers' indifference and free-entry conditions in (8) and (9) yield:

$$\begin{aligned} \lambda \psi'(p) \frac{dp}{dE} &= \left(\hat{\mu}(\theta) + E\hat{\mu}'(\theta) \frac{d\theta}{dE} \right) \phi [\psi(p) - u(b)] + \mu(\theta) \phi \psi'(p) \frac{dp}{dE}, \\ 0 &= \left(\hat{\zeta}(\theta) + E\hat{\zeta}'(\theta) \frac{d\theta}{dE} \right) p [\psi(p) - u(b)] + \zeta(\theta) [\psi(p) - u(b) + p\psi'(p)] \frac{dp}{dE}. \end{aligned}$$

The total differentiation of the indifference condition can be rearranged to read:

$$\underbrace{\frac{(\lambda - \mu(\theta)\phi) \psi'(p)}{\phi [\psi(p) - u(b)]}}_{>0 \text{ for } u(b) > 0} \frac{dp}{dE} = \hat{\mu}(\theta) + E\hat{\mu}'(\theta) \frac{d\theta}{dE} = \frac{d\mu(\theta)}{dE}, \quad (31)$$

as in any MSCC-equilibrium $\psi(p) - u(b) > 0$, and for the free entry condition we obtain:

$$-\underbrace{\frac{\zeta(\theta) [\psi(p) - u(b) + p\psi'(p)]}{p[\psi(p) - u(b)]}}_{>0} \frac{dp}{dE} = \hat{\zeta}(\theta) + E\hat{\zeta}'(\theta) \frac{d\theta}{dE} = \frac{d\zeta(\theta)}{dE}. \quad (32)$$

Hence, we obtain that the effect of E on $\mu(\theta)$ and $\zeta(\theta)$ have *opposite* signs as long as $dp/dE \neq 0$. Moreover, we can rule out $dp/dE = 0$, as this would require $d\mu(\theta)/dE = 0$ which is only possible when $d\theta/dE < 0$, but this is inconsistent with the requirement from (32) that $d\zeta(\theta)/dE = 0$ when $dp/dE = 0$. In fact, $d\theta/dE < 0 \implies d\zeta(\theta)/dE > 0$. Therefore, $dp/dE < 0$ at $u(b) = 0$ implies $dp/dE < 0$ also for $u(b) > 0$ whenever an MSCC equilibrium exists. Hence, we conclude that also $d\theta/dE < 0$ and $d\mu(\theta)/dE < 0$ from (31). Finally, $d\mu(\theta)/dE < 0$ and $dp/dE < 0$ are only consistent with more applicants and thus $dSE/dE < 0$.

■

A.4 One-worker firms

The contract posting problem is:

$$\begin{aligned} & \max_{\theta, w, l} \zeta(\theta) \lambda (p - w) l, \\ & \text{subject to: } \mu(\theta) \lambda (u(wl) - l) + (1 - \mu(\theta)) u(b) \geq V^{LM}. \end{aligned}$$

The FOCs read:

$$\frac{\partial \Pi}{\partial \theta} : \quad \zeta'(\theta) \lambda (p - w) l = \chi \mu'(\theta) [\lambda (u(wl) - l) - u(b)] \quad (33)$$

$$\frac{\partial \Pi}{\partial l} : \quad \zeta(\theta) \lambda (p - w) = \chi \mu(\theta) \lambda (u'(wl) w - 1) \quad (34)$$

$$\frac{\partial \Pi}{\partial w} : \quad -\zeta(\theta) \lambda l = \chi \mu(\theta) \lambda u'(wl) l \quad (35)$$

Combining (34) and (35) yields the familiar result:

$$u'(wl) = \frac{1}{p}, \quad (36)$$

while combining (35) and (33) yields the sharing rule:

$$\begin{aligned} \phi \lambda (p - w) l &= (1 - \phi) p [\lambda (u(wl) - l) - u(b)] \implies \\ l &= (1 - \phi) \left[u(wl) - \frac{u(b)}{\lambda} \right] + \phi \frac{wl}{p}. \end{aligned} \quad (37)$$

As in the baseline model, we have $wl = pq_s$ so that $\frac{wl}{p} = q_s$. For the free-entry condition, this implies that:

$$k = \zeta(\theta) \lambda (1 - \phi) p \left[\psi(p) - \frac{u(b)}{\lambda} \right] = \zeta(\theta) (1 - \phi) p [\lambda \psi(p) - u(b)], \quad (38)$$

and we can rewrite the expected utility of entering the labour market as:

$$V^{LM} = \lambda \mu(\theta) \phi \left[\psi(p) - \frac{u(b)}{\lambda} \right] + u(b).$$

The workers' indifference condition becomes:

$$\lambda \psi(p) = \lambda \mu(\theta) \phi \left[\psi(p) - \frac{u(b)}{\lambda} \right] + u(b) \iff \lambda \psi(p) = u(b),$$

so that (37) shows that $l = q_s$ and thus $w = p$, and (38) shows that equilibrium requires $k = 0$.

A.5 Proof of Proposition 2

Proof. Here we fix SE and abstract from the workers' indifference condition. Observe that the identity $wl = pq_s$ still holds.

a. **Effects of E .** Implicitly differentiating the goods-market clearing condition in (6):

$$\lambda(1 - SE) \left(\hat{\mu}(\theta)l + E\hat{\mu}'(\theta) \frac{d\theta}{dE}l + \mu(\theta) \frac{dl}{dp} \frac{dp}{dE} \right) = \left(\frac{dq_c}{dp} - \lambda SE \frac{dq_s}{dp} \right) \frac{dp}{dE},$$

or:

$$\lambda(1 - SE) \frac{d\mu(\theta)}{dE}l = \underbrace{\left(\frac{dq_c}{dp} - \lambda SE \frac{dq_s}{dp} - \mu(\theta) \frac{dl}{dp} \right)}_{<0} \frac{dp}{dE}. \quad (39)$$

Implicitly differentiating the free-entry condition in (9):

$$0 = \left(\hat{\zeta}(\theta) + E\hat{\zeta}'(\theta) \frac{d\theta}{dE} \right) p [\psi(p) - u(b)] + \zeta(\theta) [\psi(p) - u(b) + p\psi'(p)] \frac{dp}{dE},$$

so that:

$$\frac{d\zeta(\theta)}{dE} p [\psi(p) - u(b)] = -\zeta(\theta) [\psi(p) - u(b) + p\psi'(p)] \frac{dp}{dE}, \quad (40)$$

where $\psi(p) > u(b)$ in equilibrium and $\psi'(p) > 0$. Merging (39) and (40), we arrive add:

$$\lambda(1 - SE) \frac{d\mu(\theta)}{dE}l = -\frac{\left(\frac{dq_c}{dp} - \lambda SE \frac{dq_s}{dp} - \mu(\theta) \frac{dl}{dp} \right)}{\zeta(\theta) [\psi(p) - u(b) + p\psi'(p)]} \frac{d\zeta(\theta)}{dE} p [\psi(p) - u(b)].$$

In this case, the responses of the job-finding probability $\mu(\theta)$ and job-filling probability $\zeta(\theta)$ to increases in matching efficiency have identical signs. Increases in matching efficiency increase firm profits, so that firms open more vacancies. Consequently, labour market tightness increases, as the measure of workers searching for jobs is constant. The two effects jointly increase the job-finding probability: $\mu(\theta)$ increases. We have then also derived that overall, the job-filling probability increases. Aggregate supply rises, so that price p falls, which mitigates the increase in firm profits and market tightness. Note that it is impossible to have $d\zeta(\theta)/dE \leq 0$, because that would imply $d\theta/dE > 0$ and therefore $d\mu(\theta)/dE > 0$, contradicting identical signs.

b. **Effects of λ .** Implicitly differentiating the free-entry condition in (9):

$$\frac{d\zeta(\theta)}{d\lambda} p [\psi(p) - u(b)] = -\zeta(\theta) [\psi(p) - u(b) + p\psi'(p)] \frac{dp}{d\lambda}.$$

Therefore, the response of $\zeta(\theta)$ has the opposite sign to the response of price p to a change in λ . Implicitly differentiating the goods-market clearing condition in (6):

$$\lambda(1 - SE) \frac{d\mu(\theta)}{d\lambda}l = \underbrace{\left(\frac{dq_c}{dp} - \lambda SE \frac{dq_s}{dp} - \mu(\theta) \frac{dl}{dp} \right)}_{<0} \frac{dp}{d\lambda} - (SEq_s + (1 - SE)\mu(\theta)l).$$

These two conditions together rule out $dp/d\lambda \geq 0$ as that would imply $d\mu(\theta)/d\lambda < 0$ and $d\zeta(\theta)/d\lambda \leq 0$, which cannot hold simultaneously. Therefore, it must be that $dp/d\lambda < 0$, which implies $d\zeta(\theta)/d\lambda > 0$ and hence $d\mu(\theta)/d\lambda < 0$.

■

B Data and Estimation

In this section of the Appendix we present the definitions of the variables used in estimation, descriptive statistics of our sample, the results of the first-stage regression and the results of an alternative estimation of equation (20).

B.1 Data Sources and Definitions

- labour Market variables - we source data from ILOSTAT on the aggregate pool of employees PE , aggregate self-employment SE , aggregate number of own account workers OA and unemployment rates u for all population. We rely on unemployment rates because the data on the aggregate pool of unemployed U are only available by age (which is not available for the other labour force categories). Therefore, to construct data-counterparts of the model self-employment rate we first compute the size of the pool of unemployed U from the standard formula $u = U/(U + E) = U/(U + PE + SE)$. Then, we define our variables as follows:
 - *Self-employment rate*: The share of self-employed in the active population, $SE/(U + PE + SE)$. Next, we take the logarithm of this number.
 - *Own-account work rate*: Analogously to the self-employment rate, the share of own account workers in the active population, $OA/(U + PE + SE)$. Next, we take the logarithm of this number.
- Other variables
 - *Broadband*: We measure broadband penetration as the number of fixed-broadband subscriptions per 100 inhabitants. Fixed-broadband subscriptions refer to fixed subscriptions to high-speed access to the public Internet (a TCP/IP connection) at downstream speeds equal to, or greater than, 256 kbit/s. This includes cable modem, DSL, fibre-to-the-home/building, other fixed (wired)-broadband subscriptions, satellite broadband, and terrestrial fixed wireless broadband. This total is measured irrespective of the method of payment. It excludes subscriptions that have access to data communications (including the Internet) via mobile-cellular networks. It should include fixed WiMAX and any other fixed wireless technologies. It includes both residential subscriptions and subscriptions for organizations. *Source: ITU World Telecommunication/ICT Indicators (WTI) database 2018.*
 - *Tax Burden*: We take the tax wedge expressed in percentage points of total labour cost as an average over the tax rate for a single person without children earning the average wage, and a one-earner married couple at average earnings with two children. *Source: OECD*
 - *Replacement Rate*: We take the net replacement rate calculated for a couple with two children, both earning the average wage, for an unemployment spell of a person lasting 12 months, excluding housing benefits. *Source: OECD.*

- *Public Sector*: We take data on Government Employment (all levels) and divide it by Total Employment. *Source: ILOSTAT*.
- *GDP*: We measure GDP using the expenditure approach as GDP per capita in 2010 US Dollars, constant PPPs. Therefore, our variable precedes the change in the methodology introduced in 2019. *Source: OECD*.

B.2 Descriptive Statistics

We report the averages and average changes of all variables in our sample in Table 3. The three countries that experienced the strongest increase in the self-employment rates vary greatly in terms of the pace of broadband adoption. For the Netherlands, it grew approximately by 2 percentage points per year which is one of the largest rates in the sample. The UK places in the middle and Czech Republic at the bottom in this regard.

Table 3: Average level and average annual change (first differences) in the self-employment rate, own-account work rate, and regressors

	Self-employment		Own-account		Pred. broadband		GDP per capita		Replacement rate		Tax Burden		Public Sector	
	Level	Change	Level	Change	Level	Change	Level	Change	Level	Change	Level	Change	Level	Change
Australia	19.81	-0.10	11.61	0.01	18.76	1.58	41.57	0.59	47.43	-0.28	22.68	-0.15	16.43	-0.09
Austria	14.23	-0.10	6.97	0.01	18.36	1.55	40.84	0.46	56.50	-0.08	42.57	0.06	19.72	-0.12
Belgium	14.95	-0.15	9.24	-0.06	21.25	1.79	38.89	0.41	64.61	0.19	48.27	-0.22	-	-
Canada	16.77	-0.08	11.04	0.00	23.79	2.00	39.77	0.49	46.86	0.07	24.82	-0.43	18.90	0.01
Czech Republic	16.44	0.18	12.03	0.25	14.41	1.21	25.82	0.65	42.62	-1.57	33.85	0.14	23.80	-0.21
Denmark	9.83	-0.10	5.06	0.02	23.59	1.99	43.15	0.38	74.86	-0.13	32.00	-0.25	33.27	-0.14
Finland	13.51	-0.04	8.92	-0.06	21.14	1.78	37.39	0.47	66.46	-0.16	41.10	-0.21	-	-
France	11.39	-0.03	6.18	0.05	20.61	1.74	35.55	0.32	73.40	-0.15	45.59	-0.14	26.74	-0.12
Germany	11.64	-0.02	5.85	0.04	21.47	1.81	39.10	0.52	73.36	-0.08	42.73	-0.12	16.07	-0.21
Greece	33.51	-0.73	21.31	-0.34	18.77	1.58	26.32	0.06	43.19	0.27	40.94	0.01	21.47	0.11
Hungary	12.78	-0.30	7.34	-0.38	15.44	1.30	21.15	0.52	39.44	-0.72	44.87	-0.62	26.39	-0.46
Ireland	18.99	-0.43	12.31	-0.25	18.09	1.52	46.25	1.74	49.10	0.09	23.32	-0.19	18.19	-0.02
Italy	24.95	-0.30	16.43	-0.14	17.07	1.44	34.87	0.03	25.71	2.81	42.33	-0.00	16.10	-0.18
Japan	13.58	-0.36	7.54	-0.13	18.23	1.53	34.93	0.30	46.51	0.01	27.58	0.10	8.89	0.03
Korea	31.15	-0.64	18.20	-0.30	19.07	1.60	27.61	0.98	35.49	-0.10	18.50	0.32	8.87	0.04
Netherlands	15.25	0.29	10.31	0.32	23.45	1.97	43.75	0.51	74.01	-0.01	34.43	-0.02	16.82	-0.00
New Zealand	18.75	-0.04	11.60	0.06	18.48	1.56	30.65	0.50	47.17	-0.33	12.36	-0.25	12.47	-0.15
Norway	8.04	-0.12	5.84	-0.08	23.64	1.99	57.47	0.44	73.26	0.01	34.03	0.01	36.17	-0.06
Portugal	23.73	-0.72	17.08	-0.53	16.37	1.38	26.55	0.19	81.28	-0.57	33.28	0.08	15.38	-0.37
Spain	16.60	-0.33	10.68	-0.22	15.95	1.34	31.33	0.35	67.73	-0.46	36.36	0.06	15.83	-0.03
Sweden	10.98	-0.06	6.74	-0.04	25.11	2.11	40.18	0.66	61.38	-0.68	42.53	-0.39	29.64	-0.05
Switzerland	17.31	-0.17	8.22	-0.06	25.23	2.12	51.21	0.49	82.94	0.02	16.39	-0.08	16.58	-0.14
United Kingdom	14.57	0.15	11.14	0.19	19.71	1.66	35.99	0.45	36.25	0.03	29.66	-0.09	19.66	-0.22
United States	7.60	-0.09	4.30	-0.02	24.77	2.08	48.13	0.60	40.31	-0.16	25.03	0.02	16.63	-0.13
Average	16.52	-0.18	10.25	-0.07	20.11	1.69	37.44	0.50	56.25	-0.08	33.13	-0.10	19.89	-0.09

Notes: GDP per capita in \$1000 (level and change). All other variables are reported in percentages (for levels) and in percentage points (for changes).

B.3 First-Stage Regression

This nonlinear regression relies on the fact that roll-out of broadband predominantly relied on the copper wire of the voice telephony network or the coaxial cable of the cable TV network, so that these pre-existing infrastructures provide information on the maximum attainable broadband penetration rate (Czernich et al., 2011). Since 1998 is the first year broadband appeared in our data, we take the voice telephony and cable TV penetration rates in 1997 as instruments for the maximum reach of broadband.¹⁸ We take the values of these instruments from the OECD ICT Key Indicators and present them for each country in Table 4.

Table 4: Infrastructure penetration rates (in %)

	Cable TV, 1997	Voice Telephony, 1997	Predicted broadband, 2017
Australia	3.1	51.2	31.0
Austria	29.8	45.7	30.3
Belgium	88.7	48.5	35.1
Canada	68.4	61.6	39.2
Czech Republic	16.2	32.0	23.8
Denmark	49.5	63.6	38.9
Finland	37.6	55.6	34.9
France	9.6	57.6	34.0
Germany	50.5	55.0	35.4
Greece	0.6	51.6	31.0
Hungary	44.6	31.9	25.5
Ireland	46.9	42.1	29.8
Italy	0.2	44.9	28.2
Japan	11.3	47.9	30.1
Korea	5.9	52.0	31.5
Netherlands	93.1	56.6	38.7
New Zealand	0.3	50.5	30.5
Norway	57.5	62.6	39.0
Portugal	9.2	40.8	27.0
Spain	3.9	39.9	26.3
Sweden	60.5	68.0	41.4
Switzerland	87.5	64.5	41.6
United Kingdom	9.7	54.0	32.5
United States	64.8	66.0	40.9
Average	35.4	51.8	41.6

To arrive at a predicted broadband penetration rate, we first assume that the maximum broadband penetration rate for each country i is described by:

$$\gamma_i = \gamma_0 + \alpha_1 \text{tel_net}_{i,1997} + \alpha_2 \text{cable_net}_{i,1997}, \quad (41)$$

where $\text{tel_net}_{i,1997}$ and $\text{cable_net}_{i,1997}$ are the voice telephony and cable TV penetration rates in 1997. These γ_i enter the logistic diffusion curves that we assume to predict the broadband penetration rates B for each country i in year t , according to:

$$B_{it} = \frac{\gamma_i}{1 + e^{-\beta(t-\tau)}} + \varepsilon_{it}. \quad (42)$$

In this equation, β and τ are the coefficients for the speed and inflection point of the diffusion

¹⁸ Broadband was introduced in Canada in 1997, but we do not have access to cable TV penetration rates for 1996.

Table 5: First-stage estimation results, 1998–2017.

	Broadband Rate
γ_0	9.554*** (1.482)
α_1	0.0613*** (0.00991)
α_2	0.417*** (0.0308)
β	0.469*** (0.0214)
τ	2005.4*** (0.116)
Observations	480
R^2	0.971

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

Note: The dependent variable is the *Broadband Rate*. The sample is 24 OECD countries in years 1998–2017. Standard errors in parentheses.

process, and ε_{it} is an error term. Substituting (41) into (42), we can estimate a nonlinear first-stage regression. The coefficients of this regression are reported in Table 5. The table shows that all coefficients are significant at the 1% level, and that the fit is very good. Compared to Czernich et al. (2011), nine additional years of data result in an even larger R^2 , a slightly lower diffusion speed and an inflection point one year later. Cable TV has gained strength in predicting broadband, while the coefficient for voice telephony dropped.

We use the predicted values of this nonlinear first-stage regression in our second stage regressions. To illustrate the fit of the logistic diffusion curve, Figure 6 shows the actual and predicted broadband penetration rates for each country. The predicted broadband penetration rates in the last year of our sample are also reported in the last column of Table 4.

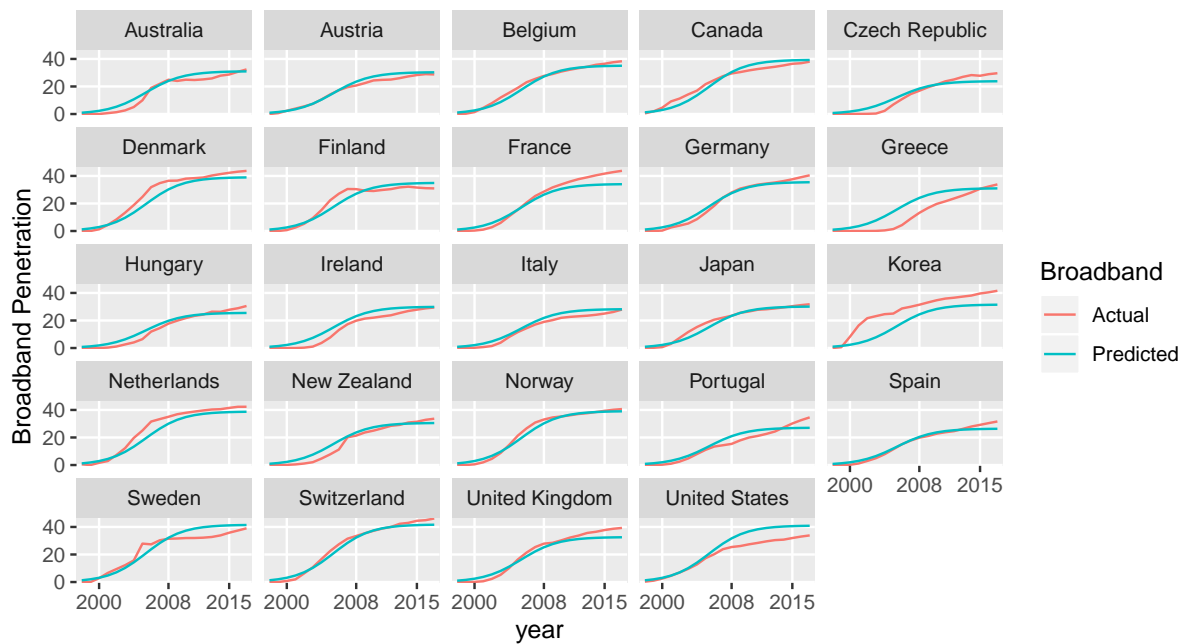


Figure 6: Actual and predicted broadband penetration rates for 22 countries, 1998–2017.

B.4 Robustness Checks

We estimate a first-differenced (FD) version of equation (20) that relies on weaker assumptions on the error term. Formally, we estimate:

$$\Delta^2 \ln SE_{it} = \beta \Delta^2 X_{it} + \omega \Delta X_{it-1} - \gamma \Delta \ln SE_{it} + \Delta \psi_t + e_{it}, \quad (43)$$

where Δ^2 is the second-difference operator. This procedure removes the unobserved fixed effects α_i at the cost of shrinking the length of the sample. We follow the Andersen-Hsiao approach and instrument the lagged dependent variable and *GDP* per capita with their previous observations. We report the results of this procedure in Tables 6 and 7.

Table 6: Effects of Broadband Internet on Self-Employment Rate: FD estimator

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate
Lagged SE rate	-0.815*** (0.076)	-0.814*** (0.076)	-0.769*** (0.081)	-0.861*** (0.121)	-0.813*** (0.076)	-0.741*** (0.130)
Δ Predicted B-band	0.007 (0.021)	0.009 (0.021)	0.017 (0.023)	0.015 (0.026)	0.021 (0.021)	0.026 (0.026)
Lagged Predicted B-band	0.013*** (0.004)	0.014*** (0.004)	0.014*** (0.005)	0.014*** (0.006)	0.015*** (0.004)	0.015*** (0.005)
Δ GDP		-0.005 (0.089)	-0.022 (0.092)	0.038 (0.095)	-0.019 (0.089)	-0.014 (0.092)
Lagged GDP		0.047 (0.091)	0.059 (0.095)	0.070 (0.101)	0.062 (0.091)	0.038 (0.099)
Δ Replacement Rate			0.004 (0.018)			0.034 (0.030)
Lagged Replacement Rate			-0.019 (0.025)			0.005 (0.043)
Δ Public Sector				-0.033 (0.046)		-0.029 (0.044)
Lagged Public Sector				-0.064 (0.057)		-0.158** (0.062)
Δ Tax Burden					0.056 (0.039)	0.074* (0.042)
Lagged Tax Burden					0.036 (0.050)	0.051 (0.055)
Observations	408	408	360	212	384	187
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	No	No	No	No	No	No
R-squared within	0.007	0.007	0.003	0.042	0.004	0.025
R-squared between	0.164	0.173	0.195	0.097	0.195	0.084
χ^2 p - value	0.000	0.000	0.000	0.000	0.000	0.000

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Effects of Broadband Internet on Own-Account Work Rate: FD estimator

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate
Lagged OA rate	-0.828*** (0.074)	-0.828*** (0.075)	-0.799*** (0.080)	-0.836*** (0.124)	-0.849*** (0.079)	-0.818*** (0.128)
Δ Predicted B-band	0.006 (0.027)	0.002 (0.027)	0.010 (0.030)	0.015 (0.033)	0.016 (0.028)	0.029 (0.035)
Lagged Predicted B-band	0.015*** (0.006)	0.015*** (0.006)	0.015*** (0.006)	0.015** (0.007)	0.017*** (0.006)	0.017** (0.007)
Δ GDP		-0.196* (0.117)	-0.209* (0.120)	-0.109 (0.122)	-0.198* (0.117)	-0.159 (0.123)
Lagged GDP		-0.132 (0.120)	-0.130 (0.123)	-0.090 (0.128)	-0.124 (0.120)	-0.141 (0.129)
Δ Replacement Rate			-0.002 (0.023)			0.032 (0.040)
Lagged Replacement Rate			-0.023 (0.032)			0.001 (0.057)
Δ Public Sector				-0.035 (0.059)		-0.038 (0.059)
Lagged Public Sector				-0.032 (0.073)		-0.150* (0.083)
Δ Tax Burden					0.015 (0.051)	0.001 (0.055)
Lagged Tax Burden					0.049 (0.065)	0.032 (0.071)
Observations	408	408	360	212	384	187
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	No	No	No	No	No	No
R-squared within	0.005	0.006	0.002	0.047	0.001	0.030
R-squared between	0.044	0.031	0.050	0.004	0.049	0.006
$\chi^2 p$ - value	0.000	0.000	0.000	0.000	0.000	0.000

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Because of lagged variables, first-differencing removes not one, but two years of data. For both dependent variables, only the long run effects of *Predicted Broadband* are significant with the same signs and interpretation as in the case of the FE estimator. The short-term effects of *Predicted Broadband* are positive, but insignificant. The negative long-run effect of *Public Sector* on the dependent variables is also present and significant. In columns (2), (3) and (6) of Table 7 we find a negative short-term effect of *GDP* on *Own Account Work* rate.

We also estimate the system of equations:

$$\Delta \ln SE_{it} = \beta \Delta X_{it} + \omega X_{it-1} - \gamma \ln SE_{it} + \alpha_i + \psi_t + \varepsilon_{it}, \quad (44)$$

$$\varepsilon_{it} = \rho \varepsilon_{it-1} + e_{it}, \quad (45)$$

allowing explicitly for serial correlation in residuals. We report the results of this exercise in Tables 8 and 9.

Table 8: Effects of Broadband Internet on Self-Employment Rate: AR(1) residuals

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate	Δ SE rate
Lagged SE rate	-0.094*** (0.021)	-0.096*** (0.021)	-0.138*** (0.027)	-0.239*** (0.039)	-0.122*** (0.025)	-0.279*** (0.048)
Δ Predicted B-band	0.025*** (0.009)	0.026*** (0.009)	0.025* (0.014)	0.047*** (0.011)	0.019 (0.012)	0.077*** (0.020)
Lagged Predicted B-band	0.003*** (0.001)	0.003*** (0.001)	0.004* (0.002)	0.008*** (0.002)	0.002 (0.002)	0.014*** (0.003)
Δ GDP		0.003 (0.083)	-0.016 (0.088)	0.003 (0.081)	0.003 (0.085)	-0.020 (0.086)
Lagged GDP		0.026 (0.030)	0.033 (0.038)	-0.021 (0.038)	0.033 (0.034)	-0.003 (0.047)
Δ Replacement Rate			0.004 (0.017)			0.040 (0.032)
Lagged Replacement Rate			-0.013 (0.013)			0.006 (0.027)
Δ Public Sector				-0.036 (0.044)		-0.055 (0.046)
Lagged Public Sector				-0.134*** (0.031)		-0.179*** (0.040)
Δ Tax Burden					0.051 (0.039)	0.062 (0.042)
Lagged Tax Burden					-0.003 (0.023)	-0.014 (0.029)
Observations	432	432	360	227	384	189
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
RSS	0.290	0.290	0.242	0.091	0.251	0.071
MSS	0.035	0.036	0.036	0.032	0.037	0.029
RMSE	0.027	0.027	0.027	0.021	0.026	0.021
R-squared within	0.108	0.110	0.131	0.263	0.128	0.292
R-squared between	0.107	0.112	0.161	0.017	0.143	0.020
AR(1) coefficient ρ	0.118	0.118	0.124	0.113	0.133	0.227
α_i share of variance	0.714	0.757	0.841	0.952	0.801	0.974
Durbin-Watson	1.795	1.795	1.795	1.814	1.776	1.668

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

We find that the results reported in the main text are robust. In the majority of the specifications we find significant and positive long- and short-term effects of *Predicted Broadband* on *Self-Employment* and *Own Account Work* rates. The size of the estimated coefficients in columns (2) in Tables 8 and 9 are very similar to the ones we report in the main text. The negative long-run effects of *Public Sector* remain negative and significant as well.

Table 9: Effects of Broadband Internet on Own-Account Work Rate: AR(1) residuals

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate	Δ OA rate
Lagged OA rate	-0.133*** (0.021)	-0.131*** (0.021)	-0.141*** (0.026)	-0.284*** (0.044)	-0.122*** (0.024)	-0.307*** (0.054)
Δ Predicted B-band	0.014 (0.011)	0.016 (0.011)	0.024 (0.017)	0.064*** (0.015)	0.019 (0.014)	0.079*** (0.026)
Lagged Predicted B-band	0.002* (0.001)	0.003** (0.001)	0.004 (0.003)	0.011*** (0.002)	0.003 (0.002)	0.013*** (0.004)
Δ GDP		-0.151 (0.109)	-0.210* (0.110)	-0.104 (0.104)	-0.183* (0.109)	-0.159 (0.113)
Lagged GDP		0.020 (0.038)	0.005 (0.045)	-0.056 (0.050)	0.005 (0.041)	-0.056 (0.060)
Δ Replacement Rate			-0.001 (0.022)			0.032 (0.043)
Lagged Replacement Rate			-0.011 (0.016)			-0.005 (0.035)
Δ Public Sector				-0.046 (0.056)		-0.075 (0.062)
Lagged Public Sector				-0.155*** (0.040)		-0.194*** (0.052)
Δ Tax Burden					0.009 (0.051)	-0.010 (0.056)
Lagged Tax Burden					0.007 (0.028)	-0.010 (0.037)
Observations	432	432	360	227	384	189
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
RSS	0.514	0.511	0.395	0.151	0.430	0.123
MSS	0.074	0.078	0.056	0.056	0.052	0.042
RMSE	0.035	0.035	0.034	0.027	0.034	0.028
R-squared within	0.127	0.132	0.123	0.272	0.108	0.253
R-squared between	0.047	0.058	0.102	0.012	0.085	0.015
AR(1) coefficient ρ	0.072	0.071	0.058	0.127	0.059	0.193
α_i share of variance	0.737	0.755	0.787	0.959	0.729	0.966
Durbin-Watson	1.874	1.876	1.905	1.777	1.902	1.710

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

C Additional Results (not for publication)

C.1 ODE representation of an MSCC equilibrium when $u(b) > 0$.

When $u(b) > 0$, it is no longer possible to separate p and θ as easily as before. However, one can solve for θ as a function of p :

$$\theta = \frac{(1 - \phi) p (\lambda \psi(p) - u(b))}{\phi k},$$

and use this equation to arrive at the equilibrium price p as an implicit function of b :

$$\lambda \psi(p) = u(b) + \mu \left(\frac{(1 - \phi) p (\lambda \psi(p) - u(b))}{\phi k} \right) \phi (\psi(p) - u(b)).$$

By differentiating this equation with respect to $u(b)$, we arrive at a non-autonomous ordinary differential equation of p in $u(b)$ with the initial condition given by p^* implied by equations (11) and (12). How far the solution $p(u(b))$ can be extended with respect to $u(b)$, yielding equilibrium $SE \in (0, 1)$, depends on the initial condition and the utility function $u(c)$ that implies in a particular $\psi(p)$.

We set $u_b = u(b)$ and differentiate the equilibrium conditions with respect to u_b .

$$\begin{aligned} \lambda \psi'(p) \frac{dp}{du_b} &= 1 + \mu'(\theta) \frac{d\theta}{du_b} \phi (\psi(p) - u_b) + \mu(\theta) \phi \left(\psi'(p) \frac{dp}{du_b} - 1 \right), \\ 0 &= \zeta'(\theta) \frac{d\theta}{du_b} p (\psi(p) - u_b) + \zeta(\theta) \left[\frac{dp}{du_b} (\psi(p) - u_b) + p \left(\psi'(p) \frac{dp}{du_b} - 1 \right) \right]. \end{aligned}$$

These ODE equations pin down the MSCC equilibria, as long as the solution to this set of equations features $SE \in (0, 1)$. For a special case of $u(c) = 2\sqrt{c}$ in which $\psi(p) = p$ (see Appendix C.2) we can simplify them as follows:

$$\begin{aligned} \lambda \frac{dp}{du_b} &= 1 + \mu'(\theta) \frac{d\theta}{du_b} \phi (p - u_b) + \mu(\theta) \phi \left(\frac{dp}{du_b} - 1 \right) \\ - \frac{\zeta'(\theta)}{\zeta(\theta)} \frac{d\theta}{du_b} p (p - u_b) &= \left[\frac{dp}{du_b} (p - u(b)) + p \left(\frac{dp}{du_b} - 1 \right) \right] \end{aligned}$$

We can sign the derivative responses to u_b at $u_b = 0$ exploiting $\lambda = \mu(\theta)\phi$ and removing u_b evaluating at $p^* = p(u(b) = 0)$ and $\theta^* = \theta(u(b) = 0)$.

$$\begin{aligned} 0 &> \frac{\mu(\theta^*)\phi - 1}{\mu'(\theta^*)\phi p^*} = \frac{d\theta}{du_b}, \\ - \frac{\zeta'(\theta^*)}{\zeta(\theta^*)} \frac{d\theta}{du_b} p^* + 1 &= 2 \frac{dp}{du_b} \iff - \frac{\zeta'(\theta^*)}{\zeta(\theta^*)} \frac{\mu(\theta^*)\phi - 1}{\mu'(\theta^*)\phi} + 1 = 2 \frac{dp}{du_b} \implies \\ 0 &> \frac{1}{1 - \phi} \left(1 - \frac{1}{\mu(\theta^*)} \right) = 2 \frac{dp}{du_b}. \end{aligned}$$

Thus, for this particular specification of $u(c) = 2\sqrt{c}$, we obtain that introducing $u(b) > 0$ initially leads to a drop in market tightness, so that the job finding probability drops and the market clearing price falls. Hence, despite smaller fraction of applicants finding jobs, the aggregate supply increases. This is only consistent with there being fewer self-employed.

C.2 Analytical solution of the model for particular matching and utility functions

Let $u(c) = 2\sqrt{c}$, $g(q) = B\frac{q^2}{2}$ and $M(A, V) = E\sqrt{AV}$. The FOC of buyers, equation (1), becomes:

$$q_c = \frac{1}{Bp^2},$$

while condition (2) is:

$$q_s = p.$$

The function $\psi(p) = u(pq_s) - q_s$ is simply a linear function of the price, $\psi(p) = p$. The identity $wl = pq_s$ implies $wl = p^2$. The matching function yields $\phi = 1/2$, $\mu(\theta) = E\sqrt{\theta}$ and $\zeta(\theta) = E/\sqrt{\theta}$. The equation that pins down the level of effort is:

$$\lambda l = (1 - \phi)(u(wl) - u(b)) + \phi \frac{wl}{p} = \frac{3}{2}p - \sqrt{b} \implies l = \frac{1}{\lambda} \left(\frac{3}{2}p - \sqrt{b} \right).$$

We are left with three equations (workers' indifference, goods market clearing, vacancy free entry condition) in p, θ, SE . Workers' indifference implies

$$\lambda p = 2\sqrt{b} + E\sqrt{\theta}\phi(p - 2\sqrt{b}) \implies \frac{2(\lambda p - 2\sqrt{b})}{E(p - 2\sqrt{b})} = \sqrt{\theta},$$

while the free-entry condition becomes:

$$k = \frac{E}{\sqrt{\theta}} \frac{p}{2} (p - 2\sqrt{b}) \implies \sqrt{\theta} = \frac{Ep}{2k} (p - 2\sqrt{b}).$$

Finally, the goods market clearing condition is:

$$SE\lambda p + (1 - SE) \frac{2(\lambda p - 2\sqrt{b})}{E(p - 2\sqrt{b})} \left(\frac{3}{2}p - \sqrt{b} \right) = \frac{1}{p^2}.$$

When $b = 0$:

$$\begin{aligned} \lambda &= E\sqrt{\theta}\phi \implies \frac{2\lambda}{E} = \sqrt{\theta}, \\ k &= \frac{E}{\sqrt{\theta}} \frac{p^2}{2} \implies p = 2 \frac{\sqrt{\lambda k}}{E}, \\ SE &= \frac{\frac{q_c}{\lambda} - \mu(\theta)l}{q_s - \mu(\theta)l} = \frac{3 - \frac{B}{p^3}}{2} = \frac{3}{2} - \frac{BE^3}{16\sqrt{\lambda^3 k^3}}. \end{aligned}$$

C.3 Congestion in the goods market

To highlight the main mechanisms, we assume in the baseline model that the probability to enter the perfectly competitive goods market is exogenous. In this section, we relax this assumption and make the probability to enter the goods market a function of the mass of prospective sellers in the goods market (the self-employed and one-worker production units) as in Rocheteau and Wright (2005). To this end, we must adjust the notation slightly. We denote E_L and θ_L as matching efficiency and tightness in the labour market, and E_G and θ_G will be their counterparts in the goods market. The two tightnesses are:

$$\theta_L = \frac{V}{A} = \frac{V}{1 - SE}, \quad (46)$$

$$\theta_G = SE + (1 - SE)\mu(\theta_L), \quad (47)$$

because the mass of buyers is normalised to 1. Just as we have $\zeta(\theta_L) = E_L \hat{\zeta}(\theta_L)$ with $\hat{\zeta}'(\theta_L) < 0$ and $\hat{\zeta}''(\theta_L) > 0$, and $\mu(\theta_L) = E_L \hat{\mu}(\theta_L)$ with $\hat{\mu}'(\theta_L) > 0$ and $\hat{\mu}''(\theta_L) < 0$, we denote $\lambda(\theta_G) = E_G \hat{\lambda}(\theta_G)$ and assume $\hat{\lambda}'(\theta_G) < 0$, $\hat{\lambda}''(\theta_G) > 0$ and $\hat{\lambda}(0) = 1$.

The decision problem of buyers is unaffected by this extension and the aggregate demand equation is still given by (1). The value of self-employment is:

$$V(q_s; p, \theta_G) = \lambda(\theta_G) (u(pq_s) - q_s),$$

but the optimal production decision is independent of $\lambda(\theta_G)$ and is again given by (2). Regarding the optimal contract-posting problem in the labour market, the firms and workers take $\lambda(\theta_G)$ as given, hence the condition (3) is unaffected while (4) and (5) are, respectively:

$$\phi(\lambda(\theta_G)p - w)l = (1 - \phi) \frac{u(wl) - \lambda(\theta_G)l - u(b)}{u'(wl)}, \quad (48)$$

$$k = \zeta(\theta_L) (\lambda(\theta_G)p - w)l. \quad (49)$$

Therefore, we are in position to define the MSCC equilibrium with congestion in the goods market.

Definition 4 (MSCC equilibrium with congestion in the goods market) A mixed-strategy career-choice equilibrium (MSCC-equilibrium) with congestion in the goods market is a tuple $(SE, p, q_s, q_c, w, l, \theta_L, \theta_G)$ such that:

- $0 < SE < 1$, $0 < \theta_L$, both types of employment are chosen in equilibrium and active in the goods market,
- given SE and θ_L , tightness in the goods market θ_G is given by (47),
- given p , each consumer demands q_c as prescribed by equation (1), each visible self-employed sells q_s given by equation (2),
- given p and θ_G , θ_L , w , l satisfy equations (3), (48) and (49),

- given $\theta_L, \theta_G, w, l, q_c$ and q_s, p and SE simultaneously clear the goods market and make workers indifferent between self-employment and searching for a job at a firm:

$$\lambda(\theta_G) (SEq_s + (1 - SE)\mu(\theta_L)l) = q_c, \quad (50)$$

$$V^{SE} = \lambda(\theta_G) (u(pq_s) - q_s) = \mu(\theta_L) (u(wl) - \lambda(\theta_G)l) + (1 - \mu(\theta_L))u(b) = V^{LM}. \quad (51)$$

Observe that the sole adjustment in the derivations leading to Lemma 1, and the results provided in this Lemma, is to replace the exogenous λ with an endogenous $\lambda(\theta_G)$. Hence, it is still the case that earnings are equalised, $wl = pq_s$ and that workers staffing visible production units produce more than the self-employed, $l > q_s$. Furthermore, for $u(b) = 0$ we have that $\lambda(\theta_G) < \mu(\theta_L)$, securing income is more likely for applicants than for the self-employed, and that $\mu(\theta_L)l > q_s$, the expected production per capita of applicants exceeds that of the self-employed. The function $\psi(p)$ is not affected by the endogeneity of $\lambda(\theta_G)$ either. Similarly, the dimensionality of the equilibrium proposed in Definition 4 can be reduced to $(\theta_G, \theta_L, SE, p)$ that satisfy the counterparts of (11) and (12):

$$\begin{aligned} \lambda(\theta_G) &= \mu(\theta_L)\phi, \\ k &= \zeta(\theta_L)(1 - \phi)p\psi(p), \end{aligned}$$

the goods-market clearing condition (51), and the definition of goods market tightness (47).

The equilibria for the baseline model can be regarded as equilibria of the model with congestion in the goods market. Let (SE^*, p^*, θ_L^*) be the solution to the baseline model for given λ, k and ϕ . Then, SE^* and θ_L^* pin down θ_G^* in the equilibrium of the model with congestion for the same k and ϕ . The only necessary adjustment is then to pick E_G such that $\lambda = \lambda(\theta_G^*)$.

Imposing Assumption 1 guarantees uniqueness of the equilibrium again. To see this, consider the thought experiment summarised in Figure 3 and in Equation (13). Suppose we start with an MSCC-equilibrium with congestion in the goods market and investigate the effects of an off-equilibrium increase in self-employment. This shift towards self-employment generates the price and labour-market tightness effects, exactly as in the baseline model, and also *increases congestion* in the goods market:

$$\frac{d}{dSE} (V^{SE} - V^{LM}) = \underbrace{\lambda'(\theta_G) \frac{d\theta_G}{dSE}}_{<0} + \underbrace{\lambda(\theta_G) \psi'(p) \frac{dp}{dSE} - \mu(\theta) \phi \psi'(p) \frac{dp}{dSE}}_{=0} - \underbrace{\mu'(\theta_L) \frac{d\theta_L}{dSE} \phi \psi(p)}_{>0} < 0.$$

Relocating a worker from the labour market to self-employment removes a fraction of $\mu(\theta_L)$ of a production unit and increases the pool of the self-employed by one worker. The resulting increase in goods market tightness makes the self-employed worse off, while large firms and applicants are not directly affected by congestion in the goods market because the equilibrium employment contract features income insurance.

Regarding the comparative statics in Theorem 2, let us differentiate workers' indifference,

free entry of vacancies and the clearing of the goods market with respect to E_G :

$$\underbrace{\hat{\lambda}(\theta_G) + \lambda'(\theta_G) \frac{d\theta_G}{dE_G}}_{\frac{d\lambda(\theta_G)}{dE_G}} = \underbrace{\mu'(\theta_L) \frac{d\theta_L}{dE_G}}_{\frac{d\mu(\theta)}{dE_G}} \phi \quad (52)$$

$$0 = \zeta'(\theta_L) \frac{d\theta_L}{dE_G} p \psi(p) + \zeta(\theta_L) [\psi(p) + p \psi'(p)] \frac{dp}{dE_G} \quad (53)$$

$$\begin{aligned} \lambda(\theta_G) \left[\frac{dSE}{dE_G} (q_s - \mu(\theta)l) + SE \frac{dq_s}{dp} \frac{dp}{dE_G} + (1 - SE) \left(\frac{d\mu(\theta)}{dE_G} l + \mu(\theta) \frac{dl}{dp} \frac{dp}{dE_G} \right) \right] \\ + \frac{d\lambda(\theta_G)}{dE_G} [SE q_s + (1 - SE) \mu(\theta)l] = \frac{dq_c}{dp} \frac{dp}{dE_G} \end{aligned} \quad (54)$$

From (52) we get that the effect of an increase in the ease of entry to the goods market must have the same sign for $\lambda(\theta_G)$ and $\mu(\theta_L)$. So, both either decrease or increase. From (53) we again obtain that price p and θ_L move in the same direction. So, higher tightness θ_L must be compensated with a higher price p . The major complication is that in the model with exogenous λ , we knew right away that $d\theta_L/d\lambda > 0$ and so $dp/d\lambda > 0$. Therefore, we have to consider all possible combinations of the effects of E_G on SE and θ_L :

1. $\frac{dSE}{dE_G} > 0, \frac{d\theta_L}{dE_G} < 0$: The latter implies that $d\lambda(\theta_G)/dE_G < 0$ because of (52). Thus, fewer workers make it to the goods market, and among those that do, per capita production decreases, because self-employment increases. Aggregate supply increases because $\mu(\theta)l > q_s$ so that prices must rise, while the free-entry condition requires the opposite to happen. Consequently, this case can be discarded.
2. $\frac{dSE}{dE_G} > 0, \frac{d\theta_L}{dE_G} > 0$: This combination we find in the baseline model. Higher E_G prompts more self-employment which increases labour market tightness. The only additional feature of the model is that the increase in SE is such that the increase in θ_G does not overcome the direct effect of E_G on $\lambda(\theta_G)$.
3. $\frac{dSE}{dE_G} < 0, \frac{d\theta_L}{dE_G} < 0$: Again, (52) requires that $\frac{d\lambda(\theta_G)}{dE_G} < 0$. However, a decrease in $\lambda(\theta_G)$ leads to a contradiction. The decrease in self-employment and drop in the job-finding probability unambiguously lead to an increase in $\lambda(\theta_G)$, on top of the direct effect of higher E_G . Consequently, this case can also be discarded.
4. $\frac{dSE}{dE_G} < 0, \frac{d\theta_L}{dE_G} > 0$: By the virtue of (52), we now must have $d\lambda(\theta_G)/dE_G > 0$ for workers' indifference. Hence, we have more workers visible in the goods market, and per worker production increases. Aggregate supply increases because $\mu(\theta)l > q_s$ so that prices should decrease, but the free-entry condition in (53) requires price to rise. Consequently, this case can be discarded as well.

Having eliminated three possible effects of an increase in E_G , we still need to consider the lack of an effect on either SE or θ_L :

1. $\frac{d\theta_L}{dE_G} = 0$: Now the free-entry condition yields that $dp/dE_G = 0$, the indifference condition implies that $d\lambda(\theta_G)/dE_G = 0$, and the goods market clearing condition then leads to

$dSE/dE_G = 0$. However, this leads to a contradiction since $d\lambda(\theta_G)/dE_G = 0$ requires $d\theta_G/dE_G > 0$, while the mass of self-employed and one-worker productive units is fixed.

2. $\frac{dSE}{dE_G} = 0, \frac{d\theta_L}{dE_G} > 0$: The indifference condition now implies that the probability to enter the goods market increases (the direct effect dominates the indirect effect). As more employees find jobs while self-employment remains constant, aggregate supply unambiguously increases, and prices fall. However, falling prices and an increase in labour market tightness violate the free-entry condition.
3. $\frac{dSE}{dE_G} = 0, \frac{d\theta_L}{dE_G} < 0$: Workers' indifference requires the probability to enter the goods market to drop. However, there are fewer one-worker production units and the same number of self-employed, resulting in a clear contradiction again.

Thus, we have arrived at $dSE/dE_G > 0$ and $d\mu(\theta_L)/dE_G > 0$ as well, exactly as in Theorem 2. For the effects of E_L , analogously to (52), we have that the sign of the effect of an increase in labour market efficiency E_L is identical for $\mu(\theta_L)$ and $\lambda(\theta_G)$:

$$\underbrace{\lambda'(\theta_G) \frac{d\theta_G}{dE_L}}_{\frac{d\lambda(\theta_G)}{dE_L}} = \underbrace{\left(\mu(\theta_L) + \mu'(\theta_L) \frac{d\theta_L}{dE_G} \right)}_{\frac{d\mu(\theta)}{dE_L}} \phi. \quad (55)$$

Now, unlike in the baseline model with $u(b) = 0$, we can rule out $d\mu(\theta)/dE_L = 0$, because condition (55) would then require that $d\lambda(\theta_G)/dE_L = 0$ as well. However, the only way to cancel the direct effect of E_L on $\mu(\theta_L)$ is via a lower self-employment rate, but as long as $\mu(\theta_L) < 1$, a lower self-employment rate unambiguously increases $\lambda(\theta_G)$. In fact, this argument also rules out $d\mu(\theta)/dE_L < 0$. Thus, the only remaining possibility is that improvements in matching efficiency in the labour market decrease the self-employment rate, $dSE/dE_L < 0$ and *increase* the job-finding probability, $d\mu(\theta)/dE_L > 0$. However, the inflow from self-employment dampens the effect of an increase in the matching efficiency in the labour market on the job-finding probability, relative to a model with a fixed self-employment rate, as in the FCC-equilibrium that we consider in the main text.

C.4 Dynamic Model

C.4.1 Model setup

Consumers Denoting the discount factor by β , consumers essentially face a static problem:

$$V^B = \max_{q_c} q_c - g(pq_c) + \beta V^B.$$

As before, the optimal demand q_c is therefore pinned down by the following equation:

$$g'(pq_c) = \frac{1}{p}. \quad (56)$$

Self-Employment We assume production cannot be stored so that the self-employed face a static problem too:

$$V^{SE} = \max_{q_s} \lambda (u(pq_s) - q_s) + \beta V^{SE},$$

with the optimal production choice q_s satisfying:

$$u'(pq_s) = \frac{1}{p}, \quad (57)$$

so that, similar to the static model,

$$(1 - \beta)V^{SE} = \lambda\psi(p)$$

with

$$\psi(p) \equiv u\left([u']^{-1}\left(\frac{1}{p}\right)\right) - \frac{1}{p}[u']^{-1}\left(\frac{1}{p}\right) = u(pq_s) - q_s.$$

Payroll Employment Production cannot be stored, and employment relationships last until destruction with probability δ . Denoting the value of being employed by V^E ,

$$\begin{aligned} V^E &= \lambda (u(wl) - l) + (1 - \lambda)u(d) + \beta (\delta V^{LM} + (1 - \delta)V^E), \\ V^E &= \frac{\lambda (u(wl) - l) + (1 - \lambda)u(d) + \beta \delta V^{LM}}{1 - \beta(1 - \delta)}, \end{aligned} \quad (58)$$

which can be substituted in the value of entering the labour market:

$$V^{LM} = \mu(\theta) V^E + (1 - \mu(\theta)) (u(b) + \beta V^{LM}).$$

Collecting terms results in the following representation:

$$V^{LM} = \frac{\mu(\theta) [\lambda (u(wl) - l) + (1 - \lambda)u(d)] + (1 - \mu(\theta)) (1 - \beta(1 - \delta)) u(b)}{(1 - \beta) [1 - (1 - \mu(\theta)) \beta(1 - \delta)]}. \quad (59)$$

Firms We anticipate perfect insurance, and substitute wl for d . Because of free entry, unfilled vacancies have no value. The value of a filled vacancy to a firm is

$$\begin{aligned} V^J &= (\lambda p - w)l + \beta(1 - \delta)V^J, \\ V^J &= \frac{(\lambda p - w)l}{1 - \beta(1 - \delta)}, \end{aligned}$$

so that the free entry condition is

$$k = \zeta(\theta)V^J = \zeta(\theta) \frac{(\lambda p - w)l}{1 - \beta(1 - \delta)}. \quad (60)$$

In the competitive search environment of the labour market, the expected value of going to a sub-market cannot be less than the market utility V^{LM} . Following the standard argument in such models with homogeneous workers and firms, there will be exactly one sub-market

active. Given p , the optimal contract chooses (θ, w, l) to solve the constrained optimisation problem to maximize per-vacancy discounted expected profits Π with respect to the market utility constraint V^{LM} :

$$\max_{\theta, w, l} \zeta(\theta) \frac{(\lambda p - w)l}{1 - \beta(1 - \delta)} \equiv \Pi$$

subject to

$$\frac{\mu(\theta) [u(wl) - \lambda l] + (1 - \mu(\theta)) (1 - \beta(1 - \delta)) u(b)}{(1 - \beta) [1 - (1 - \mu(\theta)) \beta(1 - \delta)]} \geq V^{LM}.$$

The FOCs for this problem read, with χ being the multiplier on the market utility constraint:

$$\frac{\partial \Pi}{\partial \theta} : \frac{\zeta'(\theta)(\lambda p - w)l}{1 - \beta(1 - \delta)} = \chi \mu'(\theta) \frac{u(wl) - \lambda l - (1 - \beta(1 - \delta)) u(b) - (1 - \beta)\beta(1 - \delta)V^{LM}}{(1 - \beta) [1 - (1 - \mu(\theta)) \beta(1 - \delta)]} \quad (61)$$

$$\frac{\partial \Pi}{\partial l} : \zeta(\theta) \frac{\lambda p - w}{1 - \beta(1 - \delta)} = \chi \mu(\theta) \frac{u'(wl)w - \lambda}{(1 - \beta) [1 - (1 - \mu(\theta)) \beta(1 - \delta)]} \quad (62)$$

$$\frac{\partial \Pi}{\partial w} : -\zeta(\theta) \frac{l}{1 - \beta(1 - \delta)} = \chi \mu(\theta) \frac{u'(wl)l}{(1 - \beta) [1 - (1 - \mu(\theta)) \beta(1 - \delta)]} \quad (63)$$

The FOC in (63) can be rewritten to obtain an expression for the discounted multiplier

$$\frac{\chi}{(1 - \beta) [1 - (1 - \mu(\theta)) \beta(1 - \delta)]} = \frac{-\zeta(\theta)}{\mu(\theta) u'(wl) (1 - \beta(1 - \delta))}. \quad (64)$$

Substituting (64) in (62), yields

$$\begin{aligned} \zeta(\theta) \frac{\lambda p - w}{1 - \beta(1 - \delta)} &= \frac{-\zeta(\theta) \mu(\theta) (u'(wl)w - \lambda)}{\mu(\theta) u'(wl) (1 - \beta(1 - \delta))}, \\ \lambda p - w &= \frac{\lambda - u'(wl)w}{u'(wl)}, \\ u'(wl) &= 1/p. \end{aligned} \quad (65)$$

C.4.2 Model characterization

Lemma 1

1. Combining (57) and (65) shows that, as for a static model,

$$u'(wl) = 1/p = u'(pq_s) \Leftrightarrow wl = pq_s$$

As in the static model, free entry requires $\lambda p > w$, so that $p > w$ and thus $q_s < l$.

2. Substituting (64) in (61), we can solve for the sharing rule

$$\begin{aligned}\frac{\zeta'(\theta)(\lambda p - w)l}{1 - \beta(1 - \delta)} &= \frac{-\zeta(\theta)\mu'(\theta)}{\mu(\theta)} \frac{u(wl) - \lambda l - (1 - \beta(1 - \delta))u(b) - (1 - \beta)\beta(1 - \delta)V^{LM}}{u'(wl)(1 - \beta(1 - \delta))} \\ \frac{-\zeta'(\theta)}{\zeta(\theta)}(\lambda p - w)l &= \frac{\mu'(\theta)}{\mu(\theta)} \frac{u(wl) - \lambda l - u(b) + \beta(1 - \delta)[u(b) - (1 - \beta)V^{LM}]}{u'(wl)} \quad (66) \\ \frac{-\zeta'(\theta)\theta}{\zeta(\theta)}(\lambda p - w)l &= \frac{\mu'(\theta)\theta}{\mu(\theta)} \left(1 - \frac{\beta(1 - \delta)\mu(\theta)}{1 - (1 - \mu(\theta))\beta(1 - \delta)}\right) \frac{u(wl) - \lambda l - u(b)}{u'(wl)} \\ \phi(\lambda p - w)l &= (1 - \phi) \frac{1 - \beta(1 - \delta)}{1 - (1 - \mu(\theta))\beta(1 - \delta)} \frac{u(wl) - \lambda l - u(b)}{u'(wl)} \quad (67)\end{aligned}$$

since $\phi = -\theta\zeta'(\theta)/\zeta(\theta)$ is the elasticity of the vacancy-filling probability. Using (67), we obtain an expression for expected production

$$\begin{aligned}\lambda l &= \frac{(1 - \phi)[1 - \beta(1 - \delta)](u(wl) - u(b)) + [1 - \beta(1 - \delta)(1 - \mu(\theta))]\phi wl/p}{1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)} \\ &= \frac{(1 - \phi)[1 - \beta(1 - \delta)](u(wl) - u(b) - wl/p)}{1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)} + wl/p, \quad (68) \\ &= \frac{(1 - \phi)[1 - \beta(1 - \delta)][\psi(p) - u(b)]}{1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)} + q_s. \quad (69)\end{aligned}$$

which is the dynamic equivalent of (10), showing that the expected production λl is larger than q_s , and not only just l :

Equalizing $(1 - \beta)V^{SE} = (1 - \beta)V^{LM}$, workers are indifferent if and only if

$$\lambda\psi(p) = \frac{\mu(\theta)[\lambda(u(wl) - l) + (1 - \lambda)u(d)] + (1 - \mu(\theta))(1 - \beta(1 - \delta))u(b)}{1 - (1 - \mu(\theta))\beta(1 - \delta)}$$

Now we express the RHS as a function of $\psi(p)$ too. Using (65), (66) can be rewritten and solved for expected production as a function of V^{LM}

$$\begin{aligned}\phi(\lambda p - w)l &= (1 - \phi)p[u(wl) - \lambda l - (1 - \beta(1 - \delta))u(b) - \beta(1 - \delta)(1 - \beta)V^{LM}] \\ \lambda l &= (1 - \phi)[u(wl) - (1 - \beta(1 - \delta))u(b) - \beta(1 - \delta)(1 - \beta)V^{LM}] + \phi wl/p\end{aligned}$$

Substituting λl in (59) (with $wl = d$), yields

$$\begin{aligned}V^{LM} &= \frac{\mu(\theta) \left\{ \begin{aligned} &u(wl) - \phi wl/p \\ &-(1 - \phi)[u(wl) - (1 - \beta(1 - \delta))u(b) - \beta(1 - \delta)(1 - \beta)V^{LM}] \end{aligned} \right\}}{(1 - \beta)[1 - (1 - \mu(\theta))\beta(1 - \delta)]} \\ &= \frac{(1 - \beta(1 - \delta))u(b) + \mu(\theta)\phi[u(wl) - wl/p - (1 - \beta(1 - \delta))u(b)]}{(1 - \beta)[1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)]}\end{aligned}$$

Since $wl = pq_s$, we know that $u(wl) - wl/p = \psi(p)$. Worker indifference thus holds iff

$$\lambda\psi(p) = \frac{(1 - \beta(1 - \delta))u(b) + \mu(\theta)\phi[\psi(p) - (1 - \beta(1 - \delta))u(b)]}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)}. \quad (70)$$

For future reference, note that $(1 - \beta)V^{LM}$ in (70) can be written as a weighted average of the time spent in unemployment and employment:

$$\begin{aligned}(1 - \beta)V^{LM} &= \frac{(1 - \mu(\theta)\phi)(1 - \beta(1 - \delta))u(b) + \mu(\theta)\phi\psi(p)}{(1 - \mu(\theta)\phi)(1 - \beta(1 - \delta)) + \mu(\theta)\phi}, \\ &= (1 - \omega)u(b) + \omega\psi(p),\end{aligned}$$

with $\omega \in (0, 1)$ given by

$$\omega \equiv \frac{\mu(\theta)\phi}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)}. \quad (71)$$

We express the free-entry condition as a function of $\psi(p)$ too. Substituting (68) into (60):

$$\begin{aligned}k &= \frac{\zeta(\theta)p(1 - \phi)[1 - \beta(1 - \delta)](u(wl) - u(b) - wl/p)}{(1 - \beta(1 - \delta))[1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)]} + \frac{\zeta(\theta)[p\frac{wl}{p} - wl]}{1 - \beta(1 - \delta)}, \\ &= \zeta(\theta)\frac{(1 - \phi)p[\psi(p) - u(b)]}{1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)}.\end{aligned} \quad (72)$$

Equilibrium conditions (69), (70) and (72) make up the dynamic equivalents of those in Lemma 1.

Finally, we can show that for $b = 0$, pursuing a career in payroll employment is not as risky as self-employment, in the sense that

$$\lambda < \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))}. \quad (73)$$

Worker indifference requires

$$\lambda = \frac{\mu(\theta)\phi}{\mu(\theta)\phi + (1 - \mu(\theta)\phi)[1 - \beta(1 - \delta)]} \equiv \omega.$$

Therefore, the inequality to be shown holds iff the denominator

$$\begin{aligned}\mu(\theta)\phi + (1 - \mu(\theta)\phi)[1 - \beta(1 - \delta)] &> \phi[\mu(\theta) + \delta(1 - \mu(\theta))], \\ 1 - \mu(\theta)\phi - \beta + \beta\delta + \beta\mu(\theta)\phi - \beta\delta\mu(\theta)\phi &> \phi\delta - \delta\mu(\theta)\phi, \\ 1 - \beta - (1 - \beta)\mu(\theta)\phi &> \delta[\phi - \beta - (1 - \beta)\mu(\theta)\phi],\end{aligned}$$

which is true because both $\delta \in (0, 1)$ and $\phi \in (0, 1)$. For $u(b) = 0$, we can thus fully rank quantities produced according to:

$$q_s < \lambda l < \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))}l < l.$$

Equilibrium construction With $b = 0$, we have worker indifference simply for

$$\lambda = \frac{\mu(\theta)\phi}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)}, \quad (74)$$

which pins down θ as a function of the matching function and the parameters λ , ϕ , and $\beta(1 - \delta)$. The free-entry condition simplifies to

$$k = \frac{\zeta(\theta)(1 - \phi)}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)} p\psi(p) \quad (75)$$

For a given θ , the free-entry condition can then be solved for p . Then, for fixed θ and fixed p , we can compute q_s and l .

Stocks and flows Since there is a unit square of workers, we have at any t

$$SE_t + U_t + E_t = 1.$$

End-of-period unemployment U_t is then given by

$$U_t = (1 - \mu(\theta_t)) [1 - SE_t - (1 - \delta)E_{t-1}] = (1 - \mu(\theta_t)) [U_{t-1} + \delta E_{t-1} + SE_{t-1} - SE_t],$$

where $\theta_t = V_t/A_t$ with

$$A_t = 1 - SE_t - (1 - \delta)E_{t-1} = U_{t-1} + \delta E_{t-1} - \Delta SE_t.$$

End-of-period employment E_t is given by

$$E_t = \mu(\theta_t) [1 - SE_t - (1 - \delta)E_{t-1}] + (1 - \delta)E_{t-1},$$

so that the set of state variables can be reduced to E_{t-1} . Given E_{t-1} , workers split between SE_t and A_t , and a given A_t results in U_t and E_t .

Steady state In steady state, $\Delta U_t = \Delta SE_t = 0$, so that $A = U + \delta E$, and

$$\mu(\theta) [U + \delta E] = \delta E = \zeta(\theta)V$$

Exploiting that $SE + U + E = 1$, we can characterize

$$U = \frac{\delta(1 - \mu(\theta))}{\mu(\theta) + \delta(1 - \mu(\theta))} [1 - SE] \quad (76)$$

and

$$E = \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} [1 - SE] \quad (77)$$

Market clearing in steady state thus requires

$$\lambda \left(SEq_s + (1 - SE) \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l \right) = q_c. \quad (78)$$

Theorem 2 First we characterize the comparative statics for the case in which $u(b) = 0$.

Effects of λ on θ and $\mu(\theta)$. Implicitly differentiating worker indifference in (74) with respect to

λ ,

$$1 = \underbrace{\mu'(\theta)\phi}_{>0} \underbrace{\frac{1 - \beta(1 - \delta)}{[1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)]^2}}_{>0} \frac{d\theta}{d\lambda}$$

shows that $d\theta/d\lambda > 0$, so that also $d\mu(\theta)/d\lambda > 0$.

Effect of λ on p . Define $p\psi(p) \equiv \gamma(p)$, and implicitly differentiate the free-entry condition in (75) with respect to λ

$$0 = \zeta'(\theta)(1 - \phi)\gamma(p)\frac{d\theta}{d\lambda} + \zeta(\theta)(1 - \phi)\gamma'(p)\frac{dp}{d\lambda} - k\mu'(\theta)\phi\beta(1 - \delta)\frac{d\theta}{d\lambda} \Rightarrow$$

$$\underbrace{\left[\underbrace{k\mu'(\theta)\phi\beta(1 - \delta)}_{>0} - \underbrace{\zeta'(\theta)(1 - \phi)\gamma(p)}_{<0} \right]}_{>0} \underbrace{\frac{d\theta}{d\lambda}}_{>0} = \underbrace{\zeta(\theta)(1 - \phi)\gamma'(p)}_{>0} \frac{dp}{d\lambda}$$

since Lemma 1 implies that $\gamma'(p) > 0$, so that $dp/d\lambda > 0$.

Effect of λ on SE . Note that the total differential of l with respect to λ is given by

$$\frac{dl}{d\lambda} = \frac{dl}{dp} \frac{dp}{d\lambda} + \frac{dl}{d\theta} \frac{d\theta}{d\lambda}$$

Implicitly differentiating expected production λl in (69) with respect to p

$$\lambda \frac{dl}{dp} = \underbrace{\frac{(1 - \phi)[1 - \beta(1 - \delta)]\psi'(p)}{1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)}}_{>0} + \underbrace{\frac{dq_s}{dp}}_{>0} > 0, \quad (79)$$

since supply by the self-employed responds positively to price increases, so that $dl/dp > 0$.

Implicitly differentiating expected production λl in (69) with respect to θ

$$\lambda \frac{dl}{d\theta} = \frac{(1 - \phi)[1 - \beta(1 - \delta)][\psi(p) - u(b)]\beta(1 - \delta)\phi\mu'(\theta)}{[1 - \beta(1 - \delta)(1 - \mu(\theta)\phi)]^2} > 0, \quad (80)$$

which can be understood from the increase in recruiting costs when θ increases. We can thus conclude that for $b = 0$

$$\frac{dl}{d\lambda} = \underbrace{\frac{dl}{dp}}_{>0} \underbrace{\frac{dp}{d\lambda}}_{>0} + \underbrace{\frac{dl}{d\theta}}_{>0} \underbrace{\frac{d\theta}{d\lambda}}_{>0} > 0.$$

Finally, consider the effect of λ on SE . Implicitly differentiating the market-clearing

condition in (78)

$$\underbrace{SEq_s + (1 - SE) \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l}_{>0} + \lambda \frac{dSE}{d\lambda} \left[q_s - \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l \right] + \underbrace{\lambda SE \frac{dq_s}{dp} \frac{dp}{d\lambda}}_{>0} + \underbrace{\lambda(1 - SE) \left[\frac{\delta\mu'(\theta)}{[\mu(\theta) + \delta(1 - \mu(\theta))]^2} \frac{d\theta}{d\lambda} l + \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} \frac{dl}{d\lambda} \right]}_{>0} = \underbrace{\frac{dq_c}{dp} \frac{dp}{d\lambda}}_{<0}.$$

so that

$$\frac{dSE}{d\lambda} \left[q_s - \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l \right] < 0. \quad (81)$$

We have already shown that $q_s < l\mu(\theta) / (\mu(\theta) + \delta(1 - \mu(\theta)))$ for $b = 0$, so that we can conclude that $dSE/d\lambda > 0$.

Effects of E on θ and $\mu(\theta)$. We use that $\mu(\theta) \equiv E\hat{\mu}(\theta)$, so that worker indifference is now given by

$$\lambda = \frac{\phi E \hat{\mu}(\theta)}{1 - (1 - E\hat{\mu}(\theta)\phi)\beta(1 - \delta)}$$

Implicitly differentiating worker indifference with respect to E

$$0 = \phi [1 + \beta(1 - \delta)] \left(\hat{\mu}(\theta) + E\hat{\mu}'(\theta) \frac{d\theta}{dE} \right) \implies -\underbrace{\hat{\mu}(\theta)}_{>0} = \underbrace{E\hat{\mu}'(\theta)}_{>0} \frac{d\theta}{dE}$$

shows that $d\theta/dE < 0$. Since the left-hand side is zero, and θ is the only variable that can possibly adjust, it is immediate that $d\mu(\theta)/dE = 0$.

Effects of E on p and SE . Differentiating the free-entry condition in (75) with respect to E results in

$$0 = (1 - \phi) \left[\hat{\zeta}(\theta)\gamma(p) + \zeta'(\theta)\gamma(p) \frac{d\theta}{dE} + \zeta(\theta)\gamma'(p) \frac{dp}{dE} \right] - k\beta(1 - \delta)\phi \frac{d\mu(\theta)}{dE} \implies 0 = \underbrace{\hat{\zeta}(\theta)\gamma(p)}_{>0} + \underbrace{\zeta'(\theta)\gamma(p)}_{<0} \underbrace{\frac{d\theta}{dE}}_{<0} + \underbrace{\zeta(\theta)\gamma'(p)}_{>0} \frac{dp}{dE},$$

because $d\mu(\theta)/dE = 0$, so that we conclude that $dp/dE < 0$. Analogous to above but with signs switched, the total differential of l with respect to E is given by

$$\frac{dl}{dE} = \underbrace{\frac{dl}{dp}}_{>0} \underbrace{\frac{dp}{dE}}_{<0} + \underbrace{\frac{dl}{d\theta}}_{>0} \underbrace{\frac{d\theta}{dE}}_{<0} < 0,$$

since dl/dp and $dl/d\theta$ in (79) and (80) are unchanged. Differentiating the market-clearing

condition in (78) with respect to E thus results in

$$\begin{aligned} & \lambda \frac{dSE}{dE} \underbrace{\left(q_s - \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l \right)}_{<0} + \underbrace{\lambda SE \frac{dq_s}{dp} \frac{dp}{dE}}_{<0} \\ & + \lambda(1 - SE) \left(\underbrace{\frac{\delta l}{[\mu(\theta) + \delta(1 - \mu(\theta))]^2} \frac{d\mu(\theta)}{dE}}_{=0} + \underbrace{\frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} \frac{dl}{dE}}_{<0} \right) = \underbrace{\frac{dq_c}{dp} \frac{dp}{dE}}_{>0} \end{aligned}$$

so that $dSE/dE < 0$.

Finally, we characterize the comparative statics for the case in which $u(b) > 0$

Effects of λ on p and θ . Differentiating the workers' indifference condition in (70) yields:

$$\begin{aligned} \psi(p) + \lambda \psi'(p) \frac{dp}{d\lambda} &= \frac{\mu(\theta) \phi \psi'(p)}{1 - (1 - \mu(\theta) \phi) \beta(1 - \delta)} \frac{dp}{d\lambda} + \mu'(\theta) \phi \frac{\psi(p) - (1 - \beta(1 - \delta)) u(b)}{1 - (1 - \mu(\theta) \phi) \beta(1 - \delta)} \frac{d\theta}{d\lambda} \\ &\quad - \mu'(\theta) \phi \beta(1 - \delta) \frac{(1 - \mu(\theta) \phi) (1 - \beta(1 - \delta)) u(b) + \mu(\theta) \phi \psi(p)}{[1 - (1 - \mu(\theta) \phi) \beta(1 - \delta)]^2} \frac{d\theta}{d\lambda} \implies \\ \psi(p) + \underbrace{(\lambda - \omega) \psi'(p)}_{>0 \text{ for } b>0} \frac{dp}{d\lambda} &= \underbrace{\mu'(\theta) \phi \frac{(1 - \omega \beta(1 - \delta)) [\psi(p) - u(b)]}{1 - (1 - \mu(\theta) \phi) \beta(1 - \delta)}}_{>0} \frac{d\theta}{d\lambda} \end{aligned} \quad (82)$$

using the definition of ω in (71). Note in (74) that for $b = 0$ we have $\lambda = \omega$, while (70) shows that for the mixed-strategy equilibrium to exist for $b > 0$, we need $\lambda > \omega$. Implicitly differentiating the free-entry condition in (72), we arrive at:

$$\begin{aligned} 0 &= \zeta'(\theta)(1 - \phi) p [\psi(p) - u(b)] \frac{d\theta}{d\lambda} \\ &\quad + \zeta(\theta)(1 - \phi) [\psi(p) - u(b) + p \psi'(p)] \frac{dp}{d\lambda} - k \mu'(\theta) \phi \beta(1 - \delta) \frac{d\theta}{d\lambda} \implies \\ &\quad \underbrace{\left(\frac{k \mu'(\theta) \phi \beta(1 - \delta)}{-\zeta'(\theta)(1 - \phi) p [\psi(p) - u(b)]} \right)}_{>0} \frac{d\theta}{d\lambda} = \underbrace{\zeta(\theta)(1 - \phi) [\psi(p) - u(b) + p \psi'(p)]}_{>0} \frac{dp}{d\lambda} \end{aligned} \quad (83)$$

Suppose that $dp/d\lambda = 0$. Then, (83) requires $d\theta/d\lambda = 0$ while (82) can only hold when $d\theta/d\lambda > 0$. Hence, by an argument of continuous differentiability, and using that $dp/d\lambda > 0$ at $u(b) = 0$, we can rule out $dp/d\lambda \leq 0$. Thus, $dp/d\lambda$ will not change its sign when we increase b as long as we remain in an MSCC-equilibrium, so that $dp/d\lambda > 0$. Then it is immediate from (83) that also $d\theta/d\lambda > 0$ since in any MSCC-equilibrium $\psi(p) > u(b)$.

Effect of λ on SE . Note in (79) and (80) that $dl/dp > 0$ and $dl/d\theta > 0$ do not depend on b being zero, so that the sign restriction in (81) still holds. We know that at $b = 0$ we have $dSE/d\lambda > 0$ and $q_s - l\mu(\theta) / (\mu(\theta) + \delta(1 - \mu(\theta))) < 0$. Since their product must be negative, neither of those can be equal to 0 for $b > 0$. From continuity we thus get that the same comparative statics hold for $b > 0$: the self-employment rate SE increases in λ .

Effects of E . Again, we write $\mu(\theta) = E\hat{\mu}(\theta)$ and $\zeta = E\hat{\zeta}(\theta)$, so that the workers' indifference condition in (70) yields:

$$\begin{aligned} \lambda\psi'(p)\frac{dp}{dE} &= \frac{\phi\left(\hat{\mu}(\theta) + \mu'(\theta)\frac{d\theta}{dE}\right) [\psi(p) - (1 - \beta(1 - \delta))u(b)] + \mu(\theta)\phi\psi'(p)\frac{dp}{dE}}{1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)} \\ &\quad - \phi\beta(1 - \delta)\left(\hat{\mu}(\theta) + \mu'(\theta)\frac{d\theta}{dE}\right) \frac{(1 - \mu(\theta)\phi)(1 - \beta(1 - \delta))u(b) + \mu(\theta)\phi\psi(p)}{[1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)]^2} \implies \\ &\quad \underbrace{\frac{(\lambda - \omega)\psi'(p)[1 - (1 - \mu(\theta)\phi)\beta(1 - \delta)]}{\phi(1 - \omega\beta(1 - \delta))[\psi(p) - u(b)]}}_{>0 \text{ for } u(b) > 0} \frac{dp}{dE} = \left(\hat{\mu}(\theta) + \mu'(\theta)\frac{d\theta}{dE}\right) = \frac{d\mu(\theta)}{dE} \end{aligned} \quad (84)$$

using the definition of ω in (71), as in any MSCC-equilibrium $\phi(p) - u(b) > 0$. The free-entry condition in (72) yields:

$$\begin{aligned} 0 &= \left(\hat{\zeta}(\theta) + \zeta'(\theta)\frac{d\theta}{dE}\right)(1 - \phi)p[\psi(p) - u(b)] \\ &\quad + \zeta(\theta)(1 - \phi)[\psi(p) - u(b) + p\psi'(p)]\frac{dp}{dE} - \left(\hat{\mu}(\theta) + \mu'(\theta)\frac{d\theta}{dE}\right)\phi\beta(1 - \delta)k \implies \\ \frac{d\mu(\theta)}{dE} \underbrace{\frac{\phi\beta(1 - \delta)}{1 - \phi}}_{>0} &= \frac{d\zeta(\theta)}{dE} \underbrace{p[\psi(p) - u(b)]}_{>0} + \underbrace{\zeta(\theta)[\psi(p) - u(b) + p\psi'(p)]}_{>0} \frac{dp}{dE}. \end{aligned} \quad (85)$$

Suppose that $dp/dE = 0$. Then, (84) requires $d\mu(\theta)/dE = 0$, which can only be the case for $d\theta/dE < 0$ and thus $d\zeta(\theta)/dE > 0$. However, when $dp/dE = 0$ and $d\mu(\theta)/dE = 0$, (85) can only hold for $d\zeta(\theta)/dE = 0$. Consequently, we can rule out $dp/dE = 0$, and $dp/dE < 0$ at $u(b) = 0$ implies $dp/dE < 0$ also for $u(b) > 0$ whenever an MSCC equilibrium exists. Hence, we conclude from (84) that also $d\mu(\theta)/dE < 0$, and thus $d\theta/dE < 0$ and $d\zeta(\theta)/dE > 0$. Finally, $d\mu(\theta)/dE < 0$ and $dp/dE < 0$ are only consistent with more applicants and thus $dSE/dE < 0$.